Formulas for Angles in Circles
Formed by Radii, Chords, Tangents, Secants

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Formulas for Working with Angles in Circles

(*Intercepted arcs* are arcs "cut off" or "lying between" the sides of the specified angles.)

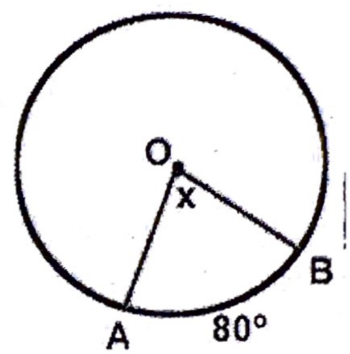
There are basically five circle formulas that you need to remember:

1. Central Angle:

A central angle is an angle formed by two intersecting radii such that its vertex is at the center of the circle.

Central Angle = Intercepted Arc

$$m\sphericalangle AOB = m\overset{\frown}{AB}$$



$<AOB$ is a central angle.
Its *intercepted arc* is the minor arc from A to B.
$m<AOB = 80°$

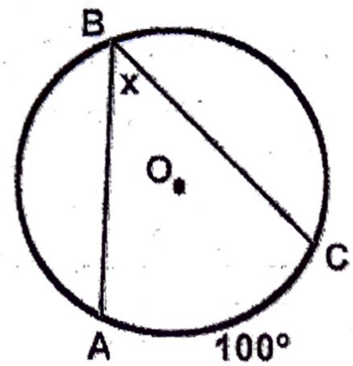
Theorem involving central angles:
In a circle, or congruent circles, congruent central angles have congruent arcs.

2. Inscribed Angle:

An inscribed angle is an angle with its vertex "on" the circle, formed by two intersecting chords.

Inscribed Angle = $\frac{1}{2}$ Intercepted Arc

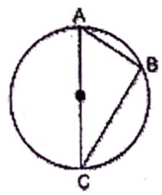
$$m\sphericalangle ABC = \frac{1}{2} m\overset{\frown}{AC}$$



$<ABC$ is an inscribed angle.
Its *intercepted arc* is the minor arc from A to C.
$m<ABC = 50°$

11.2 (2)

Special situations involving inscribed angles:

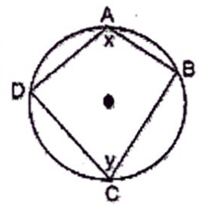


An angle inscribed in a semi-circle is a right angle.

$m\angle ABC = \frac{1}{2}\left(m\widehat{AC}\right) = \frac{1}{2}(180°) = 90°$

In a circle, inscribed circles that intercept the same arc are congruent.

A quadrilateral inscribed in a circle is called a cyclic quadrilateral.



The opposite angles in a cyclic quadrilateral are supplementary.

$m\angle x = \frac{1}{2}\left(m\widehat{DCB}\right);\ m\angle y = \frac{1}{2}\left(m\widehat{DAB}\right)$

$m\angle x + m\angle y = \frac{1}{2}\left(m\widehat{DCB} + m\widehat{DAB}\right)$

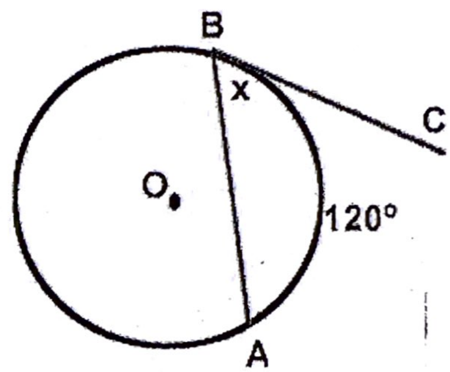
$m\angle x + m\angle y = \frac{1}{2}(360°) = 180°$

3. Tangent Chord Angle:

An angle formed by an intersecting tangent and chord has its vertex "on" the circle.

Tangent Chord Angle = $\frac{1}{2}$ Intercepted Arc

$$m\angle ABC = \frac{1}{2} m\widehat{AB}$$



<ABC is an angle formed by a tangent and chord.
Its *intercepted arc* is the minor arc from A to B.
$m\angle ABC = 60°$

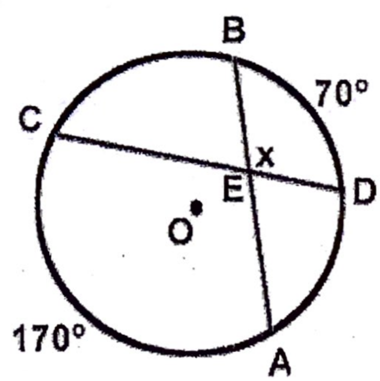
4. Angle Formed Inside of a Circle by Two Intersecting Chords:

When two chords intersect "inside" a circle, four angles are formed. At the point of intersection, two sets of vertical angles can be seen in the corners of the X that is formed on the picture. Remember: vertical angles are equal.

Angle Formed Inside by Two Chords = $\frac{1}{2}$ **Sum** of Intercepted Arcs

$$m\angle BED = \frac{1}{2}\left(m\widehat{AC} + m\widehat{BD}\right)$$

Once you have found ONE of these angles, you automatically know the sizes of the other three



<BED is formed by two intersecting chords.
Its *intercepted arcs* are $\widehat{BD}$ and $\widehat{CA}$.
[Note: the intercepted arcs belong to the set of vertical angles.]

by using your knowledge of vertical angles (being congruent) and adjacent angles forming a straight line (measures adding to 180).

$$m\angle BED = \frac{1}{2}(70+170) = \frac{1}{2}(240) = 120°$$

also, $m\angle CEA = 120°$ (vetical angle)
$m\angle BEC$ and $m\angle DEA = 60°$ by straight line.

5. Angle Formed Outside of a Circle by the Intersection of:

"Two Tangents" or "Two Secants" or "a Tangent and a Secant".

The formulas for all THREE of these situations are the same:

Angle Formed Outside = $\frac{1}{2}$ **Difference** of Intercepted Arcs

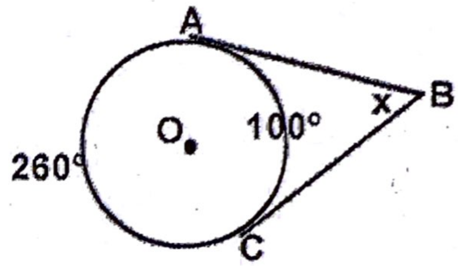
(When subtracting, start with the larger arc.)

Two Tangents:

$\angle ABC$ is formed by two tangents intersecting outside of circle *O*.

The *intercepted arcs* are minor arc $\overset{\frown}{AC}$ and major arc $\overset{\frown}{AC}$. These two arcs together comprise the entire circle.

$$m\angle ABC = \frac{1}{2}(260-100) = 80°$$



$$m\angle ABC = \frac{1}{2}\left(\overset{\text{major}}{m\overset{\frown}{AC}} - \overset{\text{minor}}{m\overset{\frown}{AC}}\right)$$

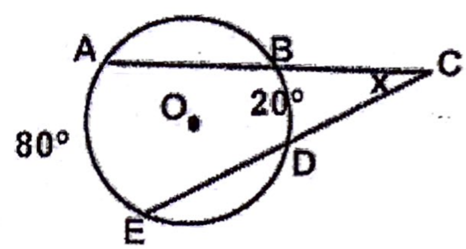
Special situation for this set up: It can be proven that $\angle ABC$ and central $\angle AOC$ are supplementary. Thus the angle formed by the two tangents and its first intercepted arc also add to 180°.

Two Secants:

$\angle ACE$ is formed by two secants intersecting outside of circle *O*.

The *intercepted arcs* are minor arcs $\overset{\frown}{BD}$ and $\overset{\frown}{AE}$.

$$m\angle ACE = \frac{1}{2}(80-20) = 30°$$



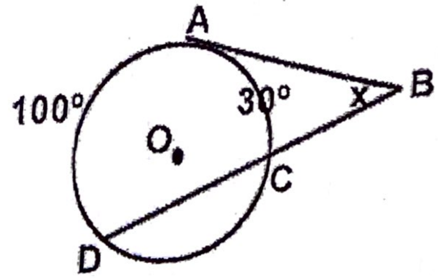
$$m\angle ACE = \frac{1}{2}\left(m\overset{\frown}{AE} - m\overset{\frown}{BD}\right)$$

a Tangent and a Secant:

$\angle ABD$ is formed by a tangent and a secant intersecting outside of circle *O*.

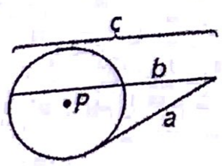
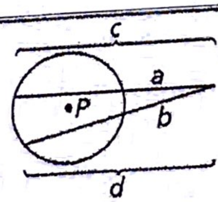
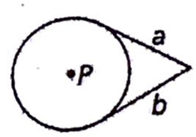
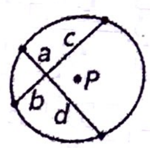
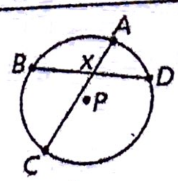
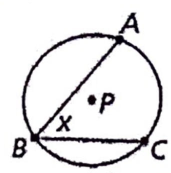
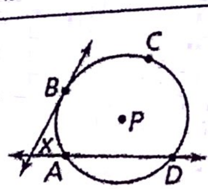
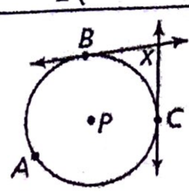
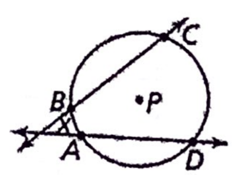
The *intercepted arcs* are minor arcs $\overset{\frown}{AC}$ and $\overset{\frown}{AD}$.

$m\angle ABD = \frac{1}{2}(100 - 30) = 35°$



$m\angle ABD = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{AC}\right)$

Circle Formulas

 $\frac{c}{a} = \frac{a}{b}$ or $a^2 = bc$	 $\frac{d}{a} = \frac{c}{b}$ or $ac = bd$	 $a = b$
 $\frac{a}{b} = \frac{c}{d}$ or $ad = bc$	 $m\angle x = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$	 $m\angle x = \frac{1}{2}m\overset{\frown}{AC}$
 $m\angle x = \frac{1}{2}\left(m\overset{\frown}{BCD} - m\overset{\frown}{AB}\right)$	 $m\angle x = \frac{1}{2}\left(m\overset{\frown}{BAC} - m\overset{\frown}{BC}\right)$	 $m\angle x = \frac{1}{2}\left(m\overset{\frown}{CD} - m\overset{\frown}{AB}\right)$