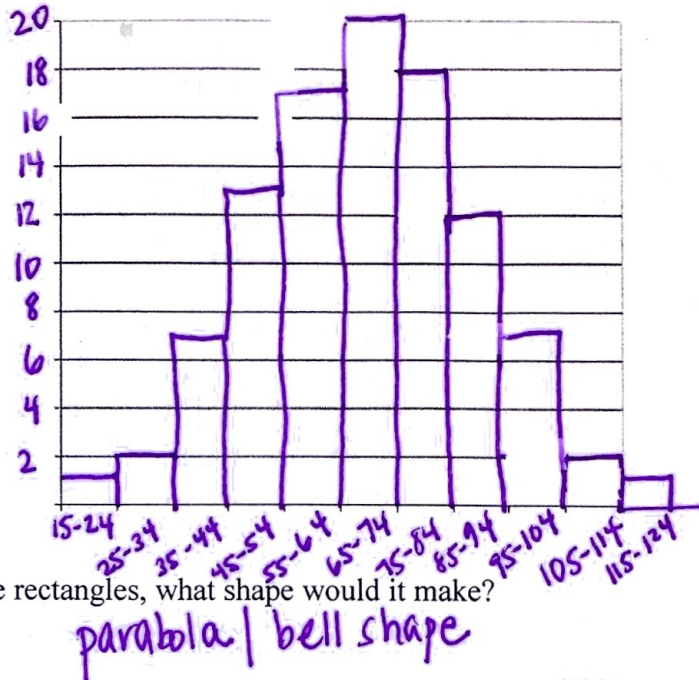


Normal Distribution

In a normal distribution, data values are concentrated around the center, with a small portion at the extreme values.

Given the frequency table below, create a histogram for the time it takes students to solve a puzzle.

Time Range (in seconds)	Number of Students
15 - 24	1
25 - 34	2
35 - 44	7
45 - 54	13
55 - 64	17
65 - 74	20
75 - 84	18
85 - 94	12
95 - 104	7
105 - 114	2
115 - 124	1



If a smooth curve is drawn through the tops of the rectangles, what shape would it make?

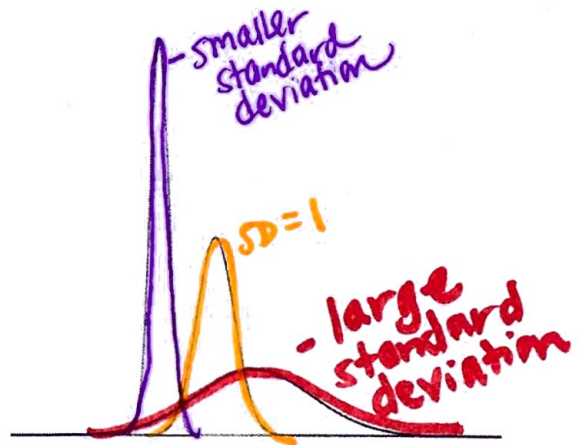
parabola / bell shape

Strictly speaking, it is not correct to talk about "the normal distribution" since there are many normal distributions. Normal distributions can differ in their means and in their standard deviations.

The figure (right) shows three normal distributions:

- The left-most distribution has a mean of -3 and a standard deviation of 0.5.
- The distribution in the middle has a mean of 0 and a standard deviation of 1.
- The distribution in right-most has a mean of 2 and a standard deviation of 3.

These as well as all other normal distributions are symmetric with relatively more values at the center of the distribution and relatively few in the tails.



Characteristics of a normal distribution:

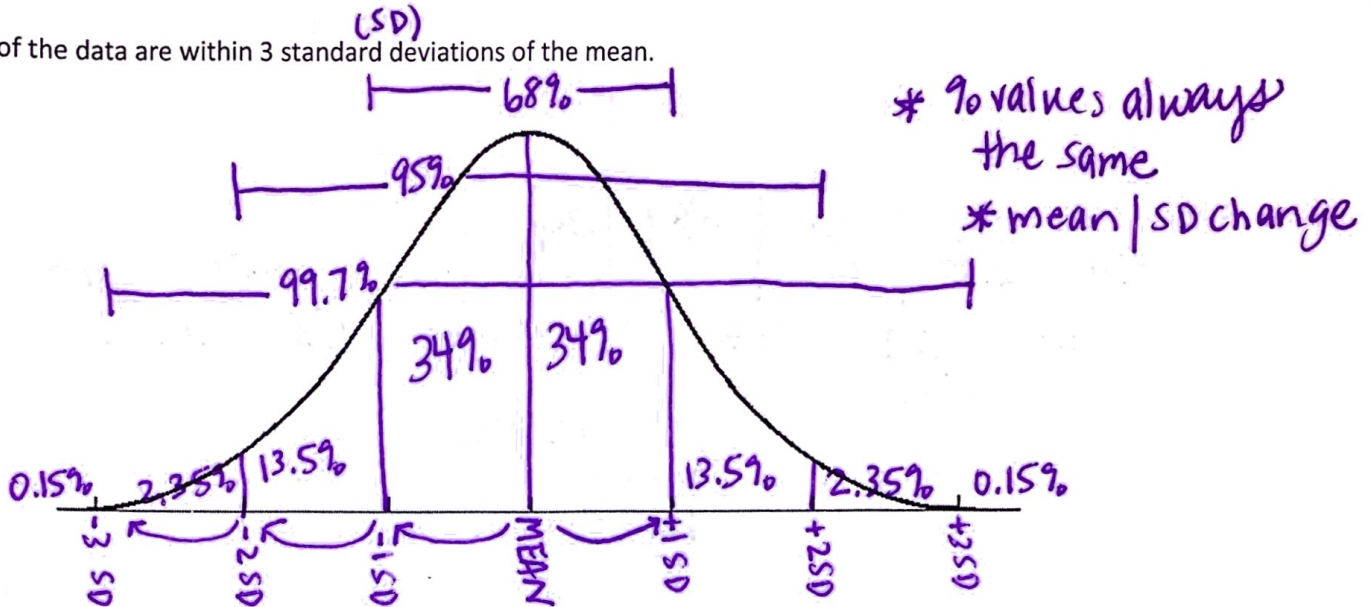
- Normal distributions are symmetric around their mean.
- The mean, median, and mode of a normal distribution are equal.
- The area under the normal curve is 1.
- Normal distributions are denser in the center and less dense in the tails.
- Normal distributions are defined by two parameters, the mean and the standard deviation.

The bell curve reaches its maximum height at the mean.

68% of the data are within 1 standard deviation of the mean.

95% of the data are within 2 standard deviations of the mean.

99.7% of the data are within 3 standard deviations of the mean.

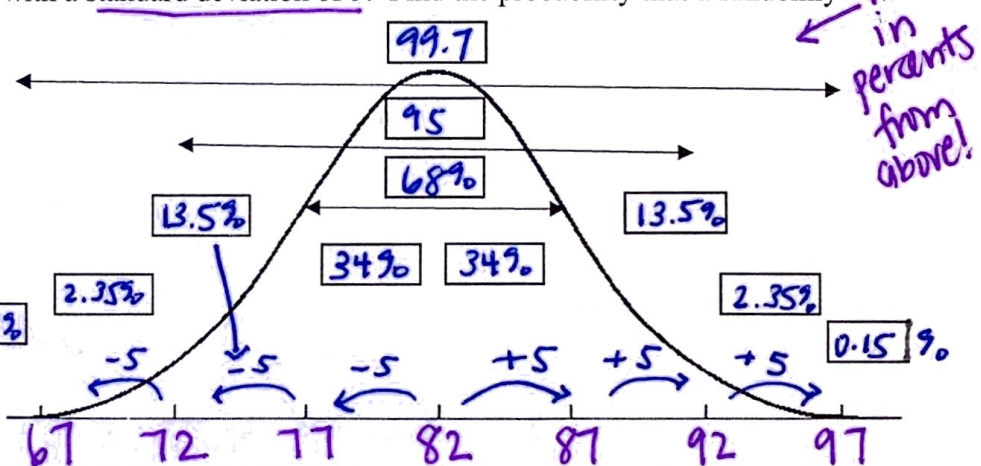


TRY!

Label the normal curve with the appropriate values using the mean (\bar{x}) and standard deviation (s). Then use your calculator to find the probability of each situation, label these in the boxes on the curve.

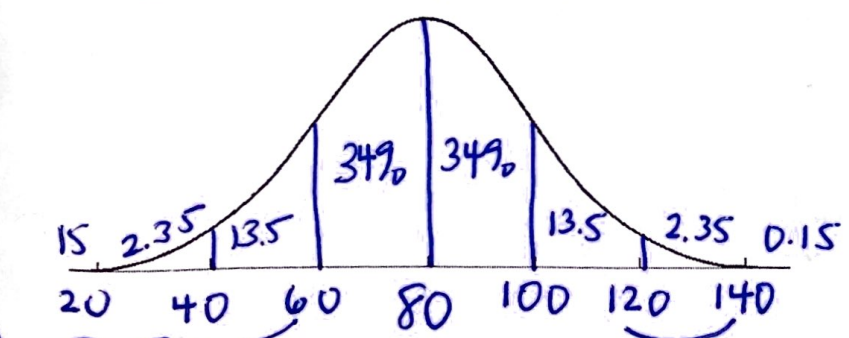
The mean score on the midterm was an 82 with a standard deviation of 5. Find the probability that a randomly selected person:

- a. scored between 77 and 87 68%
- b. scored between 72 and 92 95%
- c. scored between 67 and 97 99.7%
- d. scored less than 67 0.15%
- e. scored between 67 and 72 2.35%
- f. scored between 72 and 77 13.5%
- g. scored between 77 and 82 34%



TRY!

Example: Consider the following experiment: A psychologist asked 1000 children to solve a certain puzzle and recorded their solution times. He found the mean solving time to be 80 seconds with a standard deviation of 20 seconds. Sketch a normal curve showing the solving times one, two and three standard deviations from the mean and the % of the times.



What percent of the children had a solving time of less than 60 seconds?

$13.5 + 2.35 + 0.15 = 16\%$

Estimate what percent of the children had a solving time between 120 and 140 seconds.

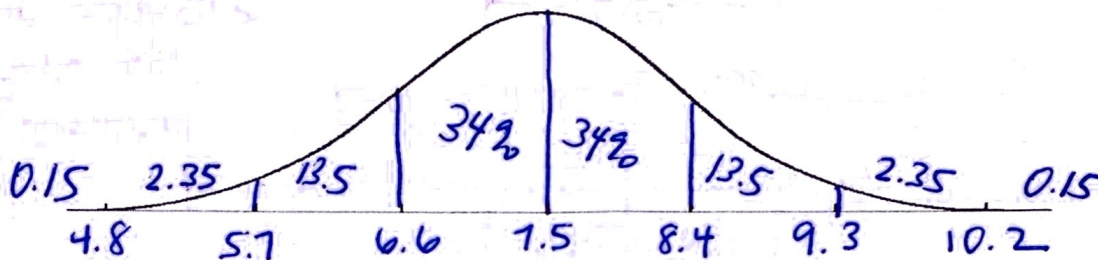
2.35%
 $(1000)(.0235) = 23.5 \rightarrow 23$

How many b/w 120-140?

Empirical Rule Practice

1. Over the last 10 years, the mean weight of newborn babies in a large hospital has been 7.5 lbs, and the standard deviation has been 0.9 lbs.

a. Sketch a normal curve showing the weights one, two and three standard deviations from the mean, and the %s of the weights.



b. Estimate what percent of the newborn babies weighed over 10.2 lbs.

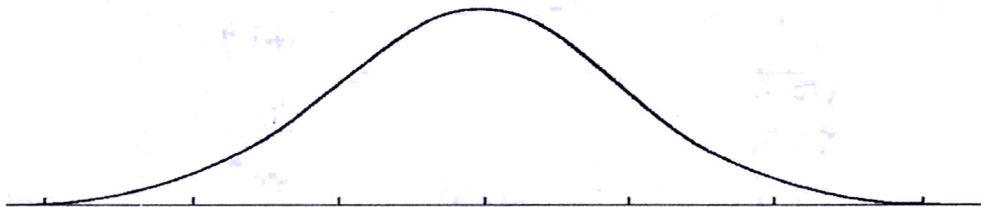
0.15%

c. Estimate what percent of the newborn babies weighed between 5.7 and 6.6 lbs.

13.5%

2. The times a group of high school students wake up on weekday mornings was found to be normally distributed. The mean wake-up time was 6:45 am and the times had a standard deviation of 15 minutes.

a. Draw the normal curve to represent the distribution of wake-up times. Identify and label each standard deviation.



b. What percent of students would you expect to wake up between 6:30 and 7:00 am?

68%

c. If 400 students were surveyed, how many would you expect to wake up between 6:00 and 7:00 am?

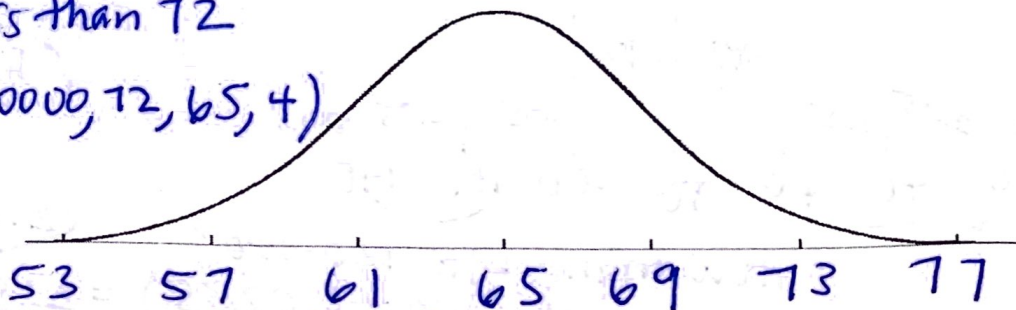
$$83.85\% \rightarrow 400(.8385) = 335.4 \rightarrow 335 \text{ students}$$

3. Suppose that to qualify for a management training program offered by your employer you must score in the top 10% of those employees who take a standardized test. Assume that the distribution of scores is normal and you received a score of 72 on the test, which had a mean of 65 and a standard deviation of 4. What percentage of those who took this test had a score below yours?

less than 72

$$\text{normalcdf}(-10000, 72, 65, 4)$$

96%



4. The mean SAT score is 490 with a standard deviation of 100. Find the probability that a randomly selected student:

a. scored between 390 and 590 68%

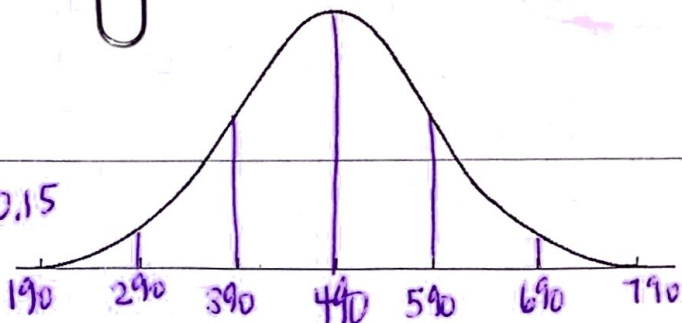
b. scored above 790 0.15%

c. scored less than 490 $34 + 13.5 + 2.35 + 0.15$

50%

d. scored between 290 and 490 $34 + 13.5$

47.5%



5. The mean weight of college football players is 200 pounds with a standard deviation of 30. Find the probability that a randomly selected player:

a. weighs between 170 and 260 $68 + 13.5$

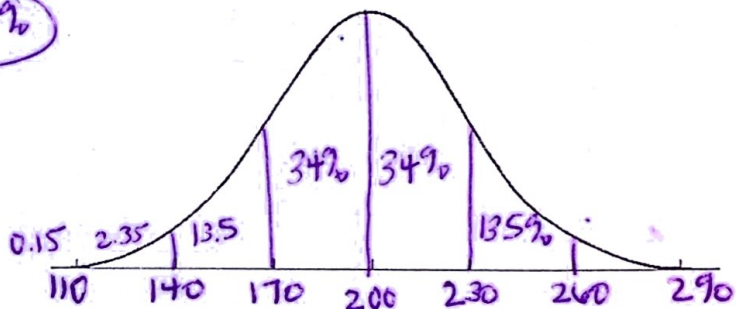
81.5%

b. weighs less than 170 16%

c. weighs over 290 0.15%

d. weighs less than 140 2.5%

e. weighs between 140 and 230 81.5%



6. The average life of a car tire is 28,000 miles with a standard deviation of 3000. Find the probability that a randomly selected tire will have a life of:

a. between 19,000 and 37,000 miles 99.77%

b. less than 25,000 miles 16%

c. between 31,000 and 37,000 miles ~~15.85%~~

16%

d. over 22,000 miles 97.5%

e. below 31,000 miles 84%

