

# Honors Math III

## Sections 8.5 – 8.7 Graphs of Polynomial Functions

In exercises 1 – 4, without using your calculator, determine the end behavior of the graph of the given polynomial functions. Then use this end behavior to match the polynomial function with its graph.

1.  $f(x) = -x^3 + 4x^2 - x$  C

R  $x \rightarrow +\infty$   $y \rightarrow -\infty$

L  $x \rightarrow -\infty$   $y \rightarrow +\infty$

3.  $f(x) = x^5 - 5x^3 + 4x$  a

R  $x \rightarrow +\infty$   $y \rightarrow +\infty$

L  $x \rightarrow -\infty$   $y \rightarrow -\infty$

2.  $f(x) = x^6 - 6x^4 + 9x^2$  b

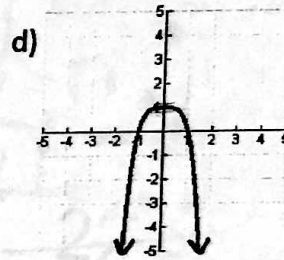
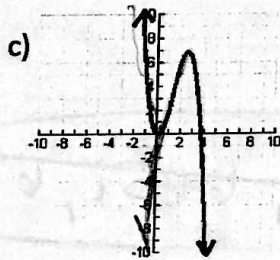
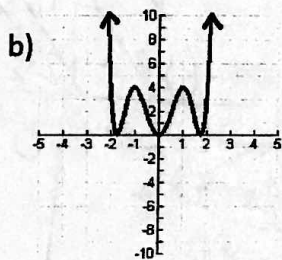
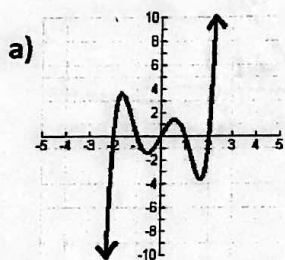
R  $x \rightarrow +\infty$   $y \rightarrow +\infty$

L  $x \rightarrow -\infty$   $y \rightarrow +\infty$

4.  $f(x) = -x^4 + 1$  d

R  $x \rightarrow +\infty$   $y \rightarrow -\infty$

L  $x \rightarrow -\infty$   $y \rightarrow -\infty$



In exercises 5 – 6, find the zeros for each polynomial function and give the multiplicity of each zero. State whether the graph crosses or touches the x-axis at each zero.

5.  $f(x) = -2(x-1)(x+2)^2(x+5)^3$

$x = 1, -2, -5$   
 dr tr

6.  $f(x) = x^3 - 5x^2 - 25x + 125$

$x = -5, 5$   
 dr

Solve and sketch the graph. Be sure to include the zeros, right and left end behavior and number of extrema.

7.  $f(x) = x^3 - x^2 - 9x + 9$

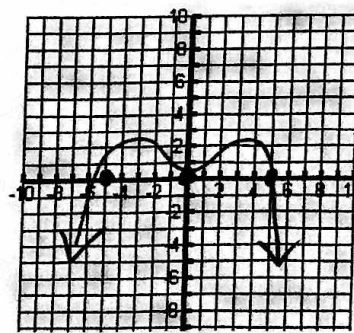
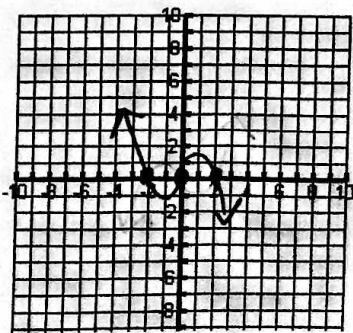
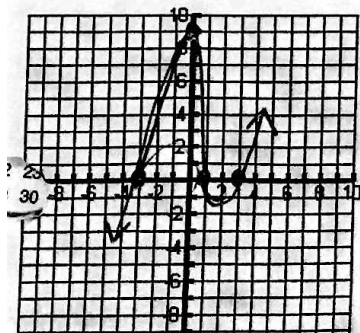
$x^2(x-1) - 9(x-1)$   
 $(x-1)(x^2-9)$   
 $(x-1)(x-3)(x+3)$   
 $x = 1, 3, -3$

8.  $f(x) = 4x - x^3$

$-x^3 + 4x = 0$   
 $-x(x^2 - 4) = 0$   
 $-x(x+2)(x-2) = 0$   
 $x = 0, x = -2, 2$

9.  $f(x) = -x^4 + 25x^2$

$-x^2(x^2 - 25)$   
 $x = 0, 5, -5$   
 dr



Use the Rational Root Theorem to list all possible rational zeros for each given function.

10.  $f(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$

$\pm 1, 5$

11.  $f(x) = 3x^5 - 2x^4 + x^2 - 12x - 7$

$f(x) = 3x^5 - 2x^4 - x^2 - 12x - 7$   
 $\frac{\pm 1, 7}{\pm 1, 3} = \pm 1, 7, \frac{1}{3}, \frac{7}{3}$

12. If  $6 + 5i$  is a root of the equation  $4x^3 - 47x^2 + 232x + 61 = 0$ , find all other solutions.

$\frac{6-5i}{12} \quad \frac{36-25i^2}{61}$

$x^2 - 12x + 61 \mid 4x^3 - 47x^2 + 232x + 61$   
 $-(4x^3 - 48x^2 + 244x)$

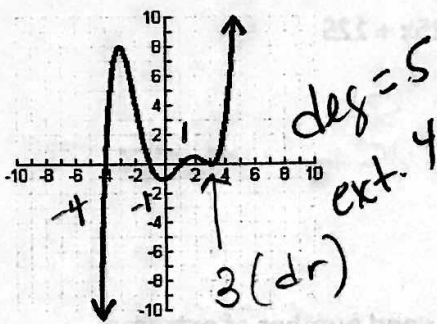
$4x + 1 = 0$

$x = -\frac{1}{4}, 6 + 5i, 6 - 5i$

$x^2 - 12x + 61$   
 $-(x^2 - 12x + 61)$   
 0

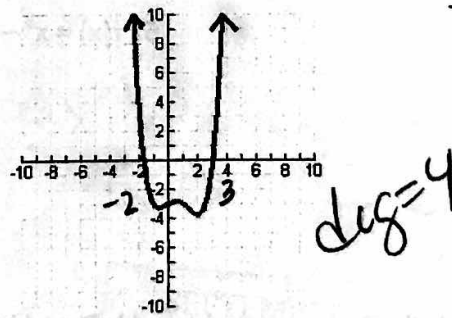
Graphs of polynomial functions are shown. In each case, specify the number of positive, negative and imaginary zeros. Indicate whether there are any real zeros with multiplicity other than 1.

13.



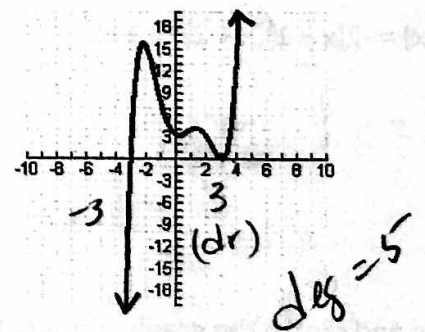
Positive: 3  
 Negative: 2  
 Imaginary: 0

14.



Positive: 1  
 Negative: 1  
 Imaginary: 2

15.



Positive: 2  
 Negative: 1  
 Imaginary: 2