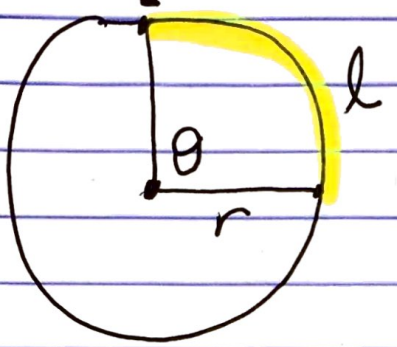
ARC LENGTH / AREA OF A SECTOR - part of area of circle



arc length - part of circumference

θ = arc measure / angle measure (degrees / radians)

IN DEGREES:

$$l = \frac{\theta}{360} \cdot 2\pi r$$

(circumference)

IN RADIANS:

$$l = \frac{\theta}{2\pi} \cdot 2\pi r$$

$$l = \theta r$$

DEGREES:

$$A = \frac{\theta}{360} \cdot \pi r^2$$

RADIANS:

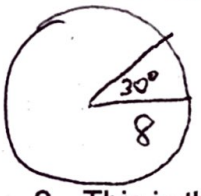
$$A = \frac{1\theta}{2\pi} \cdot \pi r^2$$

$$A = \frac{1}{2} r^2 \theta$$

Applications of Arc Length and Sectors

Guided Practice:

1. Draw a diagram of a circle with a radius of 8 units and a sector formed by a central angle of 30°. Find the arc length and area of the sector. ~~Round to the nearest tenth.~~



$\ell = \frac{30}{360} \cdot 2\pi(8) = 4.2$ units $\quad \frac{4\pi}{3}$

$A = \frac{30}{360} \cdot \pi(8)^2 = 16.8\ u^2 \quad \frac{16\pi}{3}$

2. This is the first year Janis is playing softball. She has been practicing her batting. On her last swing the bat made an arc with a radius of 48 inches and swept through 255° of rotation. Assuming the arc is circular, what is the distance the tip of the bat travels to the nearest inch? How many feet is this rounded to the nearest foot?

$\ell = \frac{255}{360} \cdot 2\pi(48) = 68\pi = 214$ in.

$\frac{214}{12} = 17.8 \rightarrow 18$ ft.

3. At Mickey's Mechanic Shop a pulley system is used to lift engines from cars. The pulley system consists of a cable that goes around a pulley with a radius 1 ft. To the nearest degree, how many degrees of rotation are required for an engine to be lifted 10 feet?

$10 = \frac{\theta}{360} \cdot 2\pi(1) \Rightarrow 10 = \frac{\pi\theta}{180}$

$\frac{1800}{\pi} = 572.96$

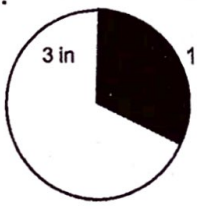
573°

Practice Problems

Use your knowledge of arc length and area of sectors to solve the following problems. Work problems on your own paper. Show all work.

1. Use the given information to find the arc length and area of each labeled sector in the following circles. Round to the nearest tenth.

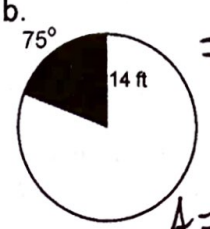
a.



$\ell = \frac{1}{3} \cdot 2\pi(3) = 6.3$ in.

$A = \frac{1}{3} \cdot \pi(3)^2 = 9.4\ in^2$

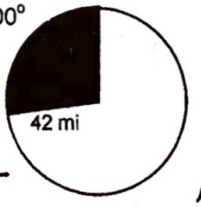
b.



$\ell = \frac{75}{360} 2\pi(14) = 18.3$ ft

$A = \frac{75}{360} \cdot \pi(14)^2 = 128.3\ ft^2$

c.



$\ell = \frac{100}{360} \cdot 2\pi(42) = 73.3$ mi

$A = \frac{100}{360} \cdot \pi(42)^2 = 1539.4\ mi^2$

$\ell = 18.8$
$A = 141.4$

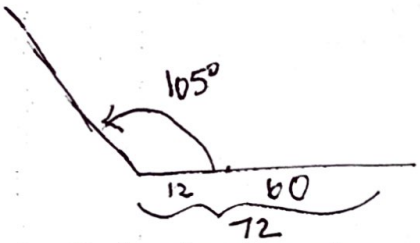
d. central angle 72°, radius 15 cm

$\ell = 26.2$
$A = 327.3$

e. central angle 60°, radius 25 yd

Honors Math 3
Applications of Arc Length and Sector Area

1. On a certain vehicle, one windshield wiper is 60 cm long, and is affixed to a swing arm which is 72 cm long from pivot point to wiper-blade tip. If the swing arm turns through 105°, what area of the windshield, to the nearest square centimeter, is swept by the wiper blade?

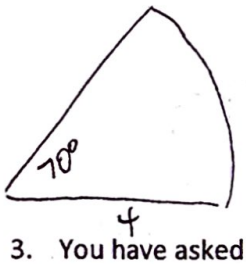


$A = \frac{105}{360} \cdot \pi(72)^2 - \frac{105}{360} \cdot \pi(12)^2$

$= 1512\pi - 42\pi = 1470\pi$

$4618\ cm^2$

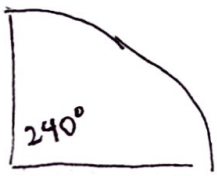
2. An adjustable-angle pop-up lawn sprinkler has been installed in an awkward corner of the neighbor's yard. This sprinkler, assuming full water pressure, can spray everything within four meters. Given that the angle has been set to 70°, how much lawn will this sprinkler head water? (Round to two decimal places.)



$A = \frac{70}{360} \cdot \pi(4)^2 = 9.77\ m^2$

3. You have asked your landscaper to create a garden area in an oddly-shaped corner of your back yard. Regarding the fencing on the two sides of the corner as being radii, the corner's central angle is two-thirds of a circle. You'd like the landscaper to install a particular type of edging, of which you have 14 2/3 feet, as the arc of a circle whose center is the pole where the two fences meet. What will be area of your new garden? (Use 22/7 for π.)

$\frac{2}{3}(360) = 240°$



$\frac{240}{360} \pi \cdot 2 \cdot r = 14\frac{2}{3}$

$\frac{4\pi}{3} r = \frac{44}{3}$

$4\pi r = 44$

$\pi r = 11$

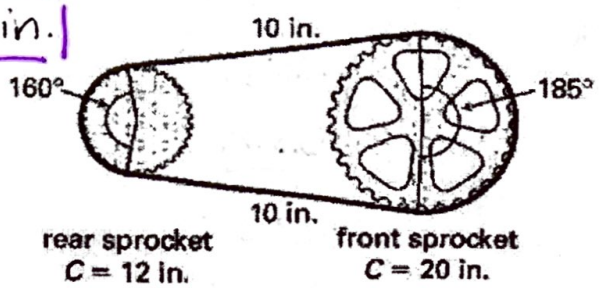
$r = \frac{11}{\pi}$

$A = \frac{2}{3}\pi\left(\frac{11}{\pi}\right)^2$

$= 25.68\ ft^2$

4. The chain of a bicycle travels along the front and rear sprockets, as shown. The circumference of each sprocket is given. About how long is the chain?

$10 + 10 + \frac{16}{3} + \frac{185}{18} = 35.61$ in.



rear $\frac{160}{360} \cdot (12) = \frac{16}{3}$

front $\frac{185}{360} \cdot 20 = \frac{185}{18}$