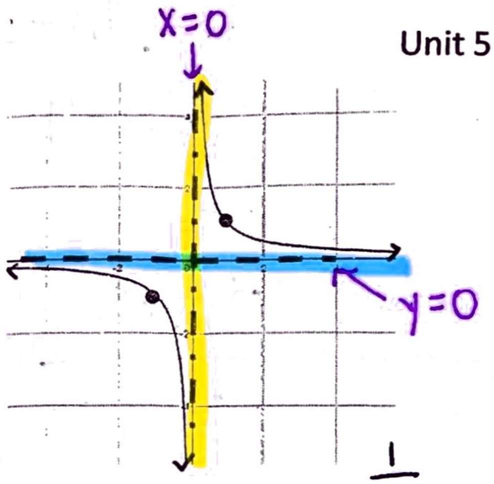
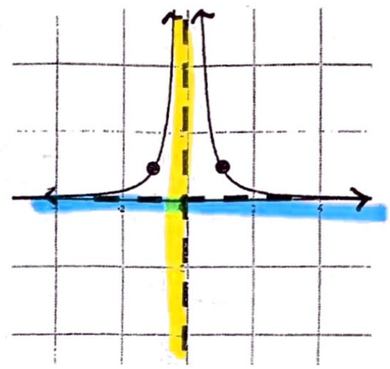
Unit 5 Graphing Rational Functions





Parent function $y = \frac{1}{x}$

Horizontal asymptote y = 0

Vertical asymptote at x = 0

HA: y=

VA: X=

Parent function $y = \frac{1}{x^2}$

Horizontal asymptote at y = 0

Vertical asymptote at x = 0

What transformations are needed to graph the following parent funtions?

Ex. 1 $y = \frac{1}{x-4} + 6$

right 4

up 6

Ex 2 $y = \frac{1}{x+1} - 5$

left 1

down 5

Ex. 3 $y = \frac{1}{(x+1)^2} + 4$

left 1

up 4

up/down

What impact does a vertical translation have on a horizontal asymptote?

moves HA up or down!

R/L

What impact does a horizontal translation have on a vertical asymptote?

moves the VA right or left

Graph the following functions transformationally. Include the 2 characteristic points and the horizontal and vertical asymptote on your graph. State the horizontal and vertical asymptotes, the domain and range.

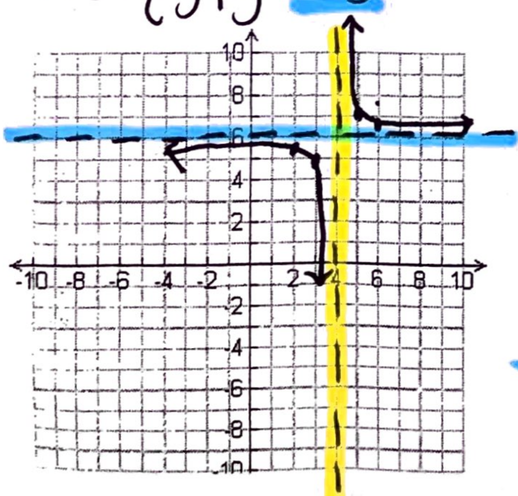
Ex. 1 $y = \frac{1}{x-4} + 6$ up 6

HA $y=6$

VA $x=4$ right 4

Domain $\{x \mid x \neq 4\}$

Range $\{y \mid y \neq 6\}$



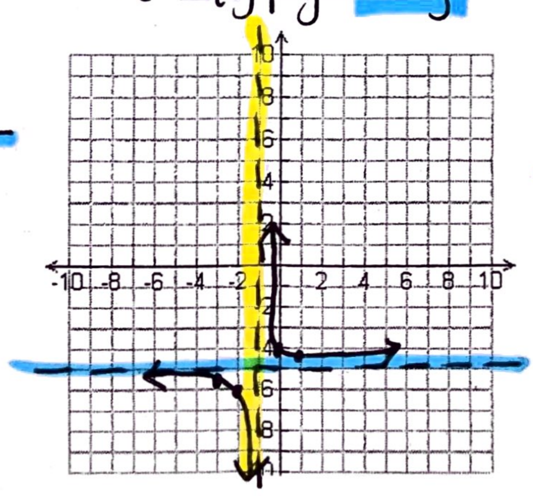
Ex 2 $y = \frac{1}{x+1} - 5$

HA $y=-5$

VA $x=-1$

Domain $\{x \mid x \neq -1\}$

Range $\{y \mid y \neq -5\}$



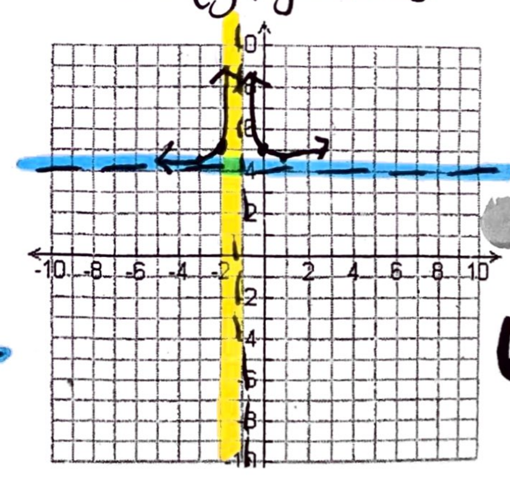
Ex. 3 $y = \frac{1}{(x+1)^2} + 4$

HA $y=4$

VA $x=-1$

Domain $\{x \mid x \neq -1\}$

Range $\{y \mid y \neq 4\}$



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Review: Simplify the following. State any restrictions on the variables.

$\frac{x+1}{x^2-1} = \frac{\cancel{x+1}}{(\cancel{x+1})(x-1)} = \boxed{\frac{1}{x-1}}$

$\frac{x^2+x-12}{x^2+7x+12} = \frac{(x-3)\cancel{(x+4)}}{(x+3)\cancel{(x+4)}} = \boxed{\frac{x-3}{x+3}}$

Vertical Asymptotes: Found by setting the factors of the denominators of a function equal to zero.

Point of Discontinuity: A hole in the graph found by setting a common factor in both the numerator and denominator equal to zero.
(hole)

FACTOR 1st!

Ex. 1: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2-1}{x^2-6x+5} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x-5)} = \frac{x+1}{x-5}$

matching factor = hole
$x-1=0$
$x=1$ $(1, -\frac{1}{2})$

$x-5=0$
$x=5 \rightarrow$ VA

$\frac{1+1}{1-5} = \frac{2}{-4} = \frac{-1}{2}$

State the domain. $\{x \mid x \neq 1, 5\}$

(anything that makes denom. = 0)

Ex. 2: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2-4}{x^2+5x+6} = \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x+3)}$

hole at
$x=2$
$x+2=0$

VA $\rightarrow x+3=0$
$x=-3$

State the domain. $\{x \mid x \neq -2, -3\}$

Horizontal Asymptotes: determined by comparing the degree of the numerator to the degree of the denominator. Let *m* = degree of numerator and *n* = degree of denominator.

If...	then.
if m < n degree in top < degree in bottom Ex. 1 $f(x) = \frac{x+1}{x^2+5x+4}$	**Then the horizontal asymptotes at y = 0** H.A.: $y=0$
If m = n degrees are = ! Ex. 2 $f(x) = \frac{x^2+5x+4}{4x^2-9}$	**Then the horizontal asymptote is y = ratio of the leading term of the numerator over the leading term of the denominator** H.A.: $y = \frac{1}{4}$
if m > n degree in top > degree in bottom Ex. 3 $f(x) = \frac{x^2-3x+4}{x-9}$	**Then there is no horizontal asymptote.**

Roots (x intercepts): Found by setting the factors of the numerator equal to zero.

Find the roots of the following rational functions.

Ex. 1 $f(x) = \frac{x+4}{x^2-x-6}$

$x+4=0$
$x=-4$
root: $(-4,0)$

Ex. 2 $f(x) = \frac{x^2+5x+4}{4x^2-9}$

$(x+4)(x+1)=0$
$x=-4, -1$
roots: $(-4,0)$ $(-1,0)$

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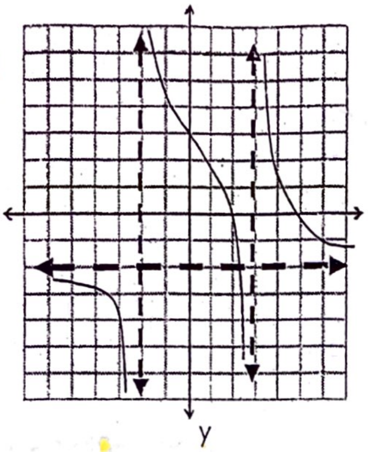
End Behavior: The behavior of the graph as ___X___ approaches either positive or negatinve infinity or as it approached a vertical asymptote.

Ex. 1



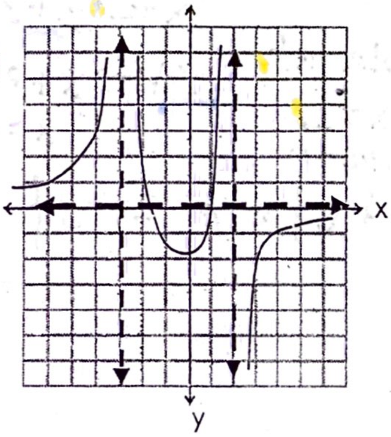
As $x \to \infty$, $y \to$ ___-2___

As $x \to -\infty$, $y \to$ ___-2___

~~As $x \to 3^+$, $y \to$ ______ but as $y \to 3^-$ → ______~~

~~As $x \to -2^+$, $y \to$ ______ but as $x \to -2^-$, $y \to$ ______~~

Ex. 2



As $x \to \infty$, $y \to$ ___0___

As $x \to -\infty$, $y \to$ ___0___

~~As $x \to 2^+$, $y \to$ ______ but as $x \to 2^-$ → ______~~

~~As $x \to -3^+$, $y \to$ ______ but as $x \to -3$, $y \to$ ______~~

Practice: State the horizontal and vertical asymptotes, roots and points of discontinuity of each equation, and then graph the function using a graphing calculator. State the domain after graphing the function completely.

Ex. 1 $f(x) = \dfrac{x-1}{x^2+x-2} = \dfrac{\cancel{x-1}}{\cancel{(x-1)}(x+2)} = \dfrac{1}{x+2}$ ← sub in $x=1$

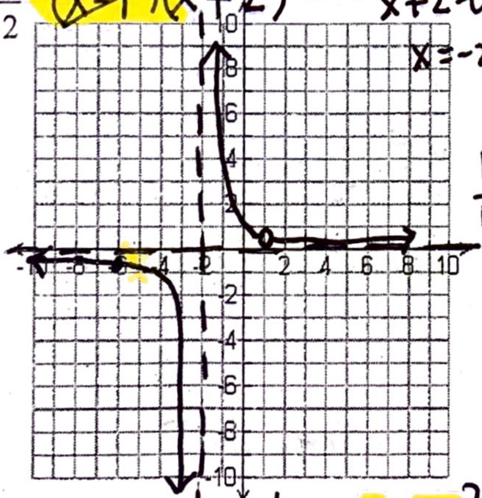
$x+2=0$

$x=-2$

Hole at $x-1=0$

$\frac{1}{1+2} = \frac{1}{3}$ $x=1$

(POINT) Hole $\left(1, \frac{1}{3}\right)$



VA $x=-2$

degree top < degree bottom

HA $y=0$

Roots ___no roots___ since num. = 1 when simplified

Domain: $\{x \mid x \neq 1, -2\}$

R As $x \to \infty$, $y \to$ ___0___

L As $x \to -\infty$, $y \to$ ___0___

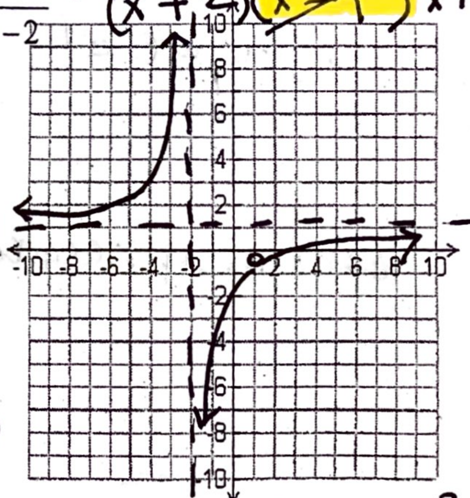
Ex. 2 $f(x) = \dfrac{1x^2-3x+2}{1x^2+x-2} = \dfrac{(x-2)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}} = \dfrac{x-2}{x+2}$

hole $x-1=0$

$x=1$

$\frac{1-2}{1+2} = \frac{-1}{3} = y$

Hole $\left(1, -\frac{1}{3}\right)$



VA $x=-2$

HA $y=1$

Roots $(2, 0)$

$x-2=0$ $x=2$

Domain: $\{x \mid x \neq 1, -2\}$

As $x \to \infty$, $y \to$ ___1___

As $x \to -\infty$, $y \to$ ___1___