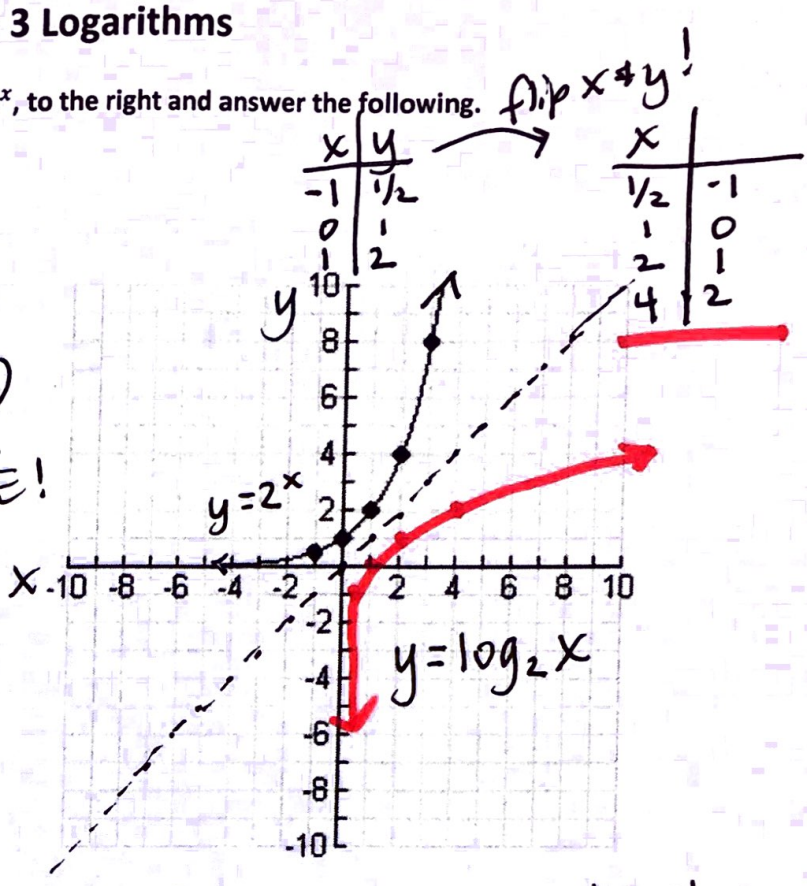
Unit 3 Logarithms

Look at the graph of the exponential function, $y = 2^x$, to the right and answer the following. flip x & y!



What is the domain? $\{x \mid x = \mathbb{R}\}$

What is the range? $\{y \mid y > 0\}$

State the horizontal asymptote. $y = 0$

What is the x intercept? none The y intercept? (0, 1)

***Sketch the line y = x and reflect the graph of $y = 2^x$.** INVERSE!

Now answer the following questions.

What is the domain? $\{x \mid x > 0\}$

What is the range? $\{y \mid y = \mathbb{R}\}$

State the vertical asymptote. $x = 0$

What is the x intercept? (1, 0) The y intercept? none

To find the equation of the reflection we need to interchange x and y and solve for y.

log always = exponent

$y = 2^x \rightarrow\rightarrow\rightarrow x = 2^y$ We need to solve this for y, but to do this we need to use logarithms.

$x = 2^y \rightarrow \log_2 x = y$

A logarithm is the inverse of an exponential function.

For the exponential function $y = b^x$, the inverse is $x = b^y$, to solve this for y, we use logarithms.

$x = b^y$ ← exponent — can be written as $\log_b(x) = y$ → base

So the inverse of $y = 2^x$ is $y = \log_2(x)$.

Write the inverse of the following functions. Remember you must first interchange x and y to find the inverse before solving for y.

a. $y = 5^x$ $x = 5^y$ $y = \log_5 x$

b. $y = 12^x$ $x = 12^y$ $y = \log_{12} x$

c. $y = 6^x$ $y = \log_6 x$

a. $y = \log_9 x$ $x = \log_9 y$ $9^x = y$

b. $y = \log_3 x$ $y = 3^x$

c. $y = \log_7 x$ $y = 7^x$

Remember: $y = b^x \longleftrightarrow x = \log_b(y)$

The base of a logarithm can be any number, just like you can use any number as the base of an exponent.

The two most frequently used numbers for the base of a logarithm are 10 and e. Because they are used more often, they can be abbreviated:

$\log_{10} x = \log x$ This is called the common logarithm or common log.

↑ understood 10!

Rewrite each equation *in logarithm form*.

1. $5^2 = 25$ → $\log_5 25 = 2$ (Exp. ↓, ↑ Base)
2. $3^{-4} = \frac{1}{81}$ → $\log_3 \frac{1}{81} = -4$
3. $6^3 = 216$ → $\log_6 216 = 3$
4. $10^4 = 10000$ → $\log_{10} 10000 = 4$ OR $\log 10000 = 4$
5. $10^{-2} = \frac{1}{100}$ → $\log_{10} \frac{1}{100} = -2$
6. $e^0 = 1$ → $\log_e 1 = 0$ OR $\ln 1 = 0$

Rewrite each equation *in exponential form*.

1. $\log_2 16 = 4$ → $2^4 = 16$
2. $\log_{16} 4 = \frac{1}{2}$ → $16^{\frac{1}{2}} = 4$
3. $\log_5 125 = 3$ → $5^3 = 125$
4. $\log_x 5 = 3$ → $x^3 = 5$
5. $\log_4 2x = 7$ → $4^7 = 2x$
6. $\log n = 2$ (base 10) → $10^2 = n$

Find the value of x in each of the following. * WRITE IN EXP. FORM! *

1. $\log_2 4 = x$ → $2^x = 4$ → $2^x = 2^2$ → $x = 2$
2. $\log_x 25 = 2$ → $x^2 = 25$ → $x = 5$
3. $\log_3 x = 3$ → $3^3 = x$ → $x = 27$

$x^{\log_x a} = a$

Evaluate the following. Hint: Set each equal to x before taking to logarithm form.

1. $2^{\log_2 5} = 5$ (base = / logs & exp. inverses!)
2. $7^{\log_7 12} = 12$
3. $10^{\log 9} = 9$ (↑ understood 10!)