INVERSE FUNCTIONS

UNIT 1
DAY 2

ONE-TO-ONE FUNCTION - Must be a function such that for every y-value there is only one x-value.

Function? yes (VLT)
One-to-one? no (HLT)

Ex. $R = \{(-2,1), (3,2), (7,-5), (-8,1)\}$

Find R^{-1} ← find the inverse of R

$R^{-1} = \{(1,-2), (2,3), (-5,7), (1,-8)\}$

INVERSE
SWITCH
x & y!

Is R a function? yes - x does not repeat
Is R^{-1} a function? no - x repeats

Is R one-to-one? no - y-values repeat
Is R^{-1} one-to-one? no - since R^{-1} is not a function

① $f(x) = 3x - 2$ LINEAR FUNCTION? yes

Find $f^{-1}(x)$.

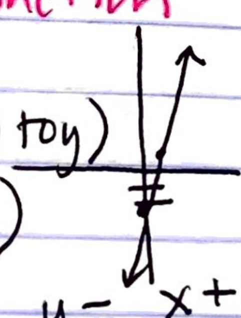
$y = 3x - 2$ (switch f(x) to y)

$x = 3y - 2$ (flip x & y)

$+2 \quad\quad +2$ (solve for y)

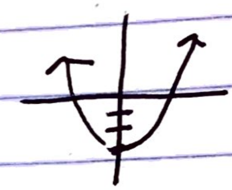
$\frac{x+2}{3} = \frac{3y}{3}$

$y = \frac{x+2}{3}$



$$f^{-1}(x) = \frac{x+2}{3} = \frac{1}{3}x + \frac{2}{3}$$ Linear

QUADRATIC: Function ✓
One-to-one ✗

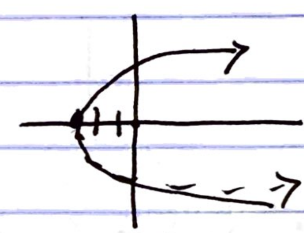


② $g(x) = x^2 - 3 \rightarrow$ find $g^{-1}(x)$

$$y = x^2 - 3$$

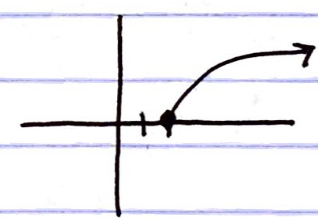
$$x = y^2 - 3$$
$$+3 \qquad +3$$
$$\sqrt{x+3} = \sqrt{y^2}$$

$$\boxed{\pm\sqrt{x+3} = y = g^{-1}(x)}$$



FUNCTION ✗
ONE-TO-ONE ✗

③ $h(x) = \sqrt{x-2}$



SQU. ROOT
FUNCTION ✓
ONE-TO-ONE ✓
D: $\{x \mid x \geq 2\}$
R: $\{y \mid y \geq 0\}$

Find $h^{-1}(x)$

$$y = \sqrt{x-2}$$
$$x = \sqrt{y-2} \quad ^2$$

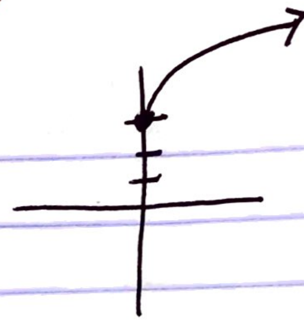
$$x^2 = y - \cancel{2}$$
$$+2 \qquad +2$$

$$x^2 + 2 = y = h^{-1}(x)$$

D: $\{x \mid x \geq 0\}$
R: $\{y \mid y \geq 2\}$

$$h^{-1}(x) = x^2 + 2, \; x \geq 0$$

④ $g(x) = \sqrt{x} + 3$



$D: \{x \mid x \geq 0\}$

$R: \{y \mid y \geq 3\}$

$y = \sqrt{x} + 3$

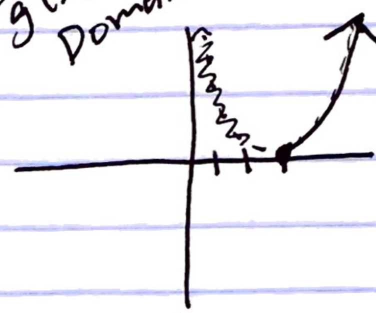
$x = \sqrt{y} + 3$

$(x-3)^2 = (\sqrt{y})^2$

$(x-3)^2 = y$

$g^{-1}(x) = (x-3)^2, \ x \geq 3$

Flip R from g(x) to get Domain of $g^{-1}(x)$



⑤ $g(x) = \sqrt{x} + 3$ $\qquad g^{-1}(x) = (x-3)^2$

$$g(g^{-1}(x)) = g(\boxed{(x-3)^2}) = \sqrt{(x-3)^2} + 3$$
$$= x - \cancel{3} + \cancel{3}$$
$$= \boxed{x}$$

$$g^{-1}(g(x)) = g^{-1}(\boxed{\sqrt{x} + 3}) = (\sqrt{x} + \cancel{3} - \cancel{3})^2$$
$$= (\sqrt{x})^2 = \boxed{x}$$

⑥ $f(x) = \dfrac{x^5 - 3}{2}$ $\quad g(x) = (2x+3)^{1/5}$

Show $f(x)$ & $g(x)$ are inverses by using composition.

$$f(g(x)) = f\left((2x+3)^{1/5}\right)$$

$$= \frac{\left(\left(2x+3\right)^{\cancel{1/5}}\right)^{\cancel{5}} - 3}{2}$$

$$= \frac{2x+\cancel{3}-\cancel{3}}{2}$$

$$= \frac{\cancel{2}x}{\cancel{2}} = x \checkmark$$

$$g(f(x)) = g\left(\frac{x^5-3}{2}\right) = \left(\cancel{2}\left(\frac{x^5-3}{\cancel{2}}\right)+3\right)^{1/5}$$

$$= \left(x^5\cancel{-3}\cancel{+3}\right)^{1/5}$$

$$= \left(x^5\right)^{1/5} = x \checkmark$$

so $f(x)$ & $g(x)$ are inverses!