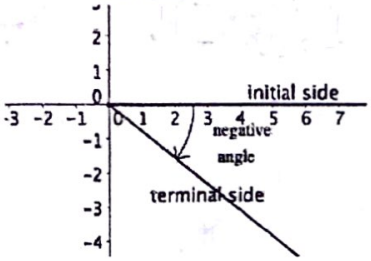
Math III Investigation: Reference and Coterminal Angles

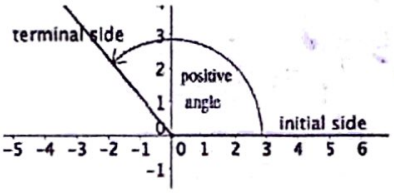
Trigonometry: the branch of mathematics that studies relationships involving lengths and angles of triangles.

Angle: generated by the rotation of two rays that share a fixed endpoint

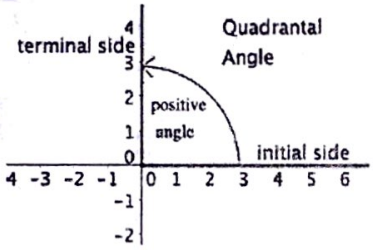
- *Initial side:* where angle begins
- *Terminal side:* where angle ends
- Positive angle – counterclockwise (CCW)
- Negative angle – clockwise (CW)

An angle with its vertex at the origin and its initial side along the positive x-axis is said to be in **standard position.**





If the terminal side of an angle in standard position coincides with one of the axes, the angle is called a **quadrantal angle.** 0°, 90°, 180°, 270°, 360°



Using your sectors: How many $\frac{\pi}{6}$ sectors are needed to complete one entire circle? 12

How many $\frac{\pi}{4}$ sectors are needed? 8 How many $\frac{\pi}{3}$ sectors? 6

30° = $\frac{\pi}{6}$ r 45° = $\frac{\pi}{4}$ r 60° = $\frac{\pi}{3}$ r

Using the colored sectors in your envelope, construct the following angles in standard position. What quadrant does the terminal side lie in?

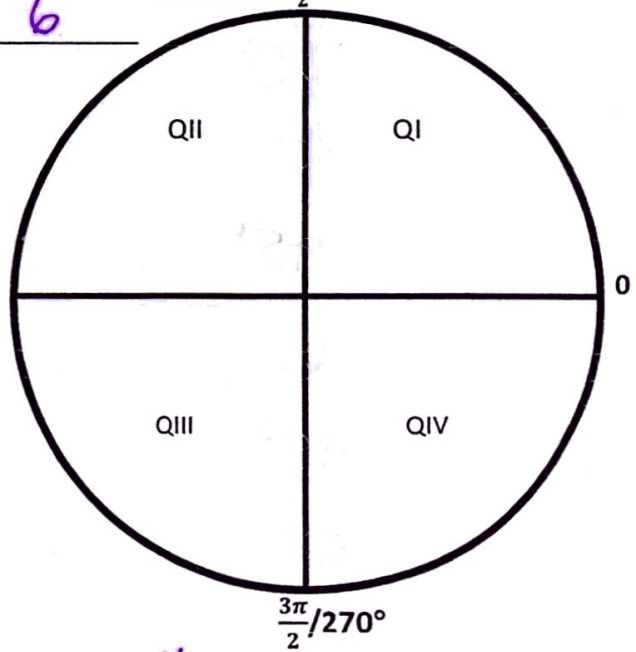
a) $\frac{2\pi}{3}$ II b) $\frac{7\pi}{6}$ III

c) 225° III d) 120° II

e) $-\frac{\pi}{3}$ IV f) $-\frac{11\pi}{6}$ I

g) −210° II h) − 60° IV

$\frac{5\pi}{6}$ II j) $\frac{5\pi}{3}$ IV



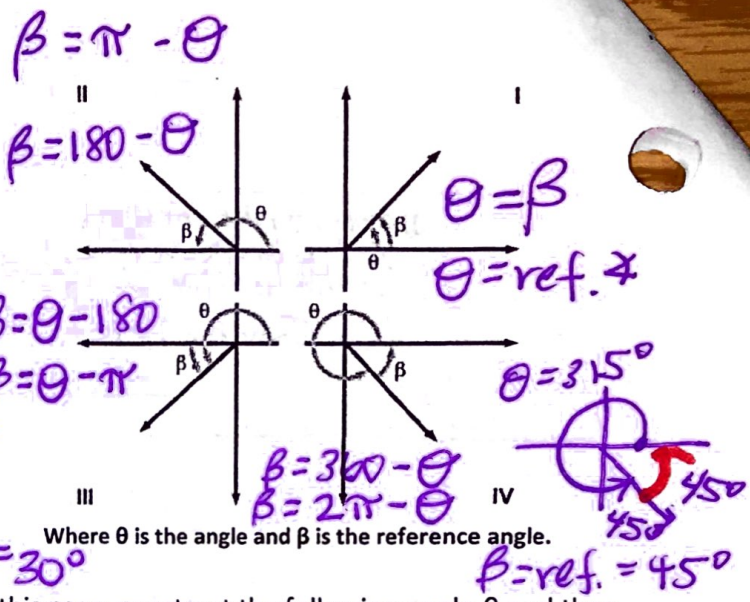
Construct $\frac{\pi}{2}$ using your sectors. Where does the terminal side lie? What about π or $\frac{3\pi}{2}$? These angles are called quadrantal angles. Why do you think that is? y-axis x y

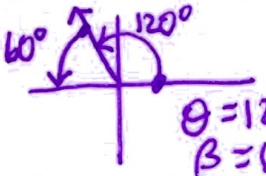
terminal side = x or y axis

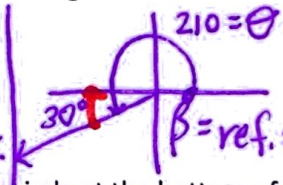
Reference Angles:

Associated with every angle drawn in standard position (except for quadrantal angles) there is another angle called the reference angle. The reference angle is the acute angle formed by the terminal side of the given angle and the axis. The reference angle is always a positive, acute angle. !!

Note that angles in quadrant I are their own reference angles and quadrantal angles do not have reference angles.







Where θ is the angle and β is the reference angle.

Using the sectors in your envelope and the circle at the bottom of this page, construct the following angle θ and then determine the reference angle β. Consider the quadrant the terminal side of θ lies in and the diagram above.

Working with radian measure:

a) $\theta = \frac{3\pi}{4}$ β= $\pi/4$ b) $\theta = \frac{7\pi}{6}$ β= $\frac{\pi}{6}$ c) $\theta = \frac{4\pi}{3}$ β= $\frac{\pi}{3}$

d) $\theta = \frac{\pi}{3}$ β= $\pi/3$ e) $\theta = \frac{11\pi}{6}$ β= $\frac{\pi}{6}$ f) $\theta = \frac{2\pi}{3}$ β= $\frac{\pi}{3}$

g) $\theta = -\frac{2\pi}{3}$ β= $\frac{\pi}{3}$ h) $\theta = -\frac{7\pi}{6}$ β= $\frac{\pi}{6}$ i) $\theta = -\frac{7\pi}{4}$ β= $\frac{\pi}{4}$

k) $\theta = \frac{\pi}{6}$ β= $\frac{\pi}{6}$ l) $\theta = -\frac{\pi}{4}$ β= $\frac{\pi}{4}$ m) $\theta = -\frac{5\pi}{3}$ β= $\frac{\pi}{3}$

Working with degree measure:

a) θ = 120° β = 60° b) θ = 225° β = 45°

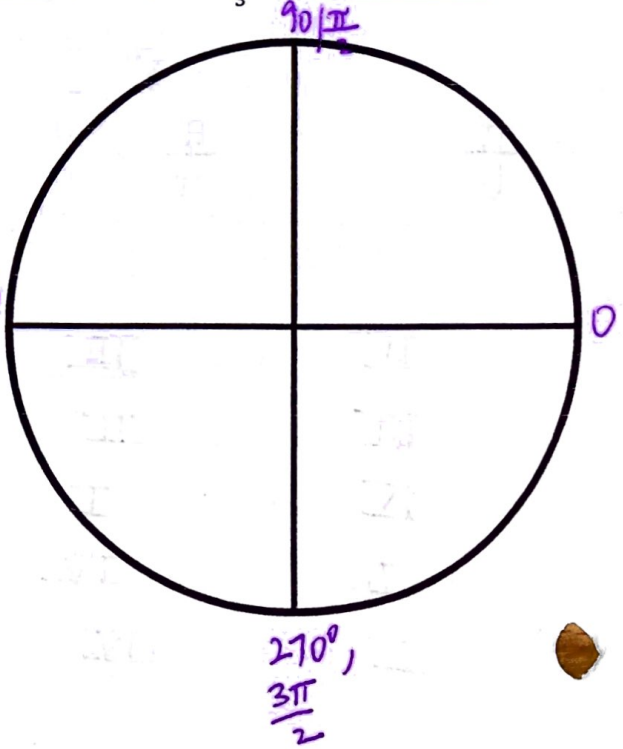
c) θ = 210° β = 30° d) θ = 300° β = 60°

e) θ = −45° β = 45° f) θ = 30° β = 30°

g) θ = -240° β = 60° h) θ = 135° β = 45°

i) θ = 45° β = 45° i) θ = −210° β = 30°

j) θ = 330° β = 30° k) θ = 150° β = 30°



Coterminal Angles:

Two angles in standard position are called coterminal angles if they have the same terminal side. Angles with the same terminal side differ only in the number of revolutions. Angles in standard positons that differ in radian measure by a multiple of 2π or that differ in degree measure by $360°$ are coterminal angles. Every angle has an infinite number of coterminal angles. Why do you think this is?

Using the circle at the bottom, construct the angle $\frac{\pi}{3}$. If you wanted to describe a negative angle in standard position with the with the same terminal side as $\frac{\pi}{3}$, what angle would you use? $-5\pi/3$ $6\pi/3$

Remember that $2\pi = \frac{12\pi}{6} = \frac{8\pi}{4} = \frac{6\pi}{3} = \frac{4\pi}{2}$. Sketch an angle that would be 2π greater than $\frac{\pi}{3}$. What angle is this? $\frac{7\pi}{3}$

To find coterminal angles, we must add or subtract multiples of 2π if in radian measure or multiples of $360°$ in degrees. Find a positive and negative coterminal angle for the following sets.

Working with radian measure: $\pm 2\pi$

12π/6 a) $\frac{\pi}{6}$ $\frac{13\pi}{6}$, $\frac{-11\pi}{6}$ 8π/4 b) $\frac{3\pi}{4}$ $\frac{11\pi}{4}$, $\frac{-5\pi}{4}$

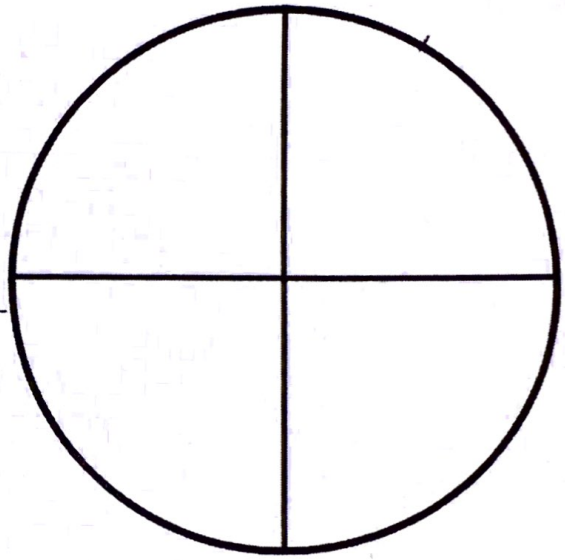
6π/4 c) $\frac{5\pi}{4}$ $\frac{13\pi}{4}$, $\frac{-3\pi}{4}$ 6π/3 d) $\frac{2\pi}{3}$ $\frac{8\pi}{3}$, $\frac{-4\pi}{3}$

d)

6π/3 e) $-\frac{\pi}{3}$ $\frac{5\pi}{3}$, $\frac{-7\pi}{3}$ 12π/6 f) $-\frac{5\pi}{6}$ $\frac{7\pi}{6}$, $\frac{-17\pi}{6}$

4π/2 g) $\frac{5\pi}{2}$ $\frac{9\pi}{2}$, $\frac{-3\pi}{2}$ 2π h) 3π 5π, $-\pi$ π

6π/3 i) $\frac{7\pi}{3}$ $\frac{13\pi}{3}$, $\frac{-5\pi}{3}$ 6π/3 j) $-\frac{8\pi}{3}$ $\frac{-2\pi}{3}$, $\frac{4\pi}{3}$



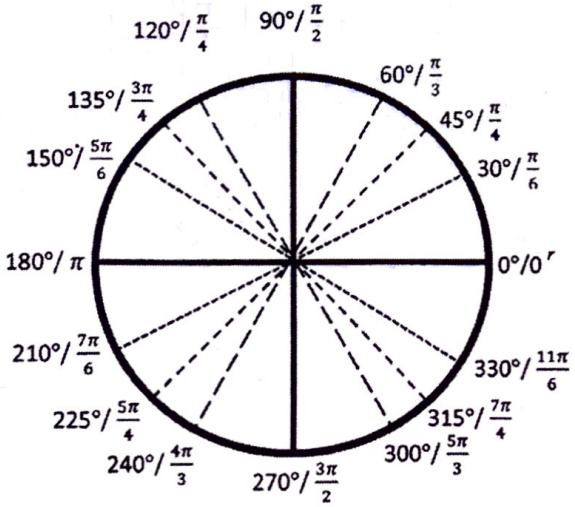
Working with degree measure: ± 360

a) $45°$ $405°$, $-315°$ b) $150°$ $510°$, $-210°$

c) $240°$ $600°$, $-120°$ d) $-60°$ $300°$, $-420°$ e) $-210°$ $150°$, $-570°$

f) $-135°$ $225°$, $-495°$ g) $390°$ $-330°$, $30°$ h) $540°$ $-180°$, $180°$

750° 900°



You will need to memorize the angles in this circle.

UNIT CIRCLE

Later...

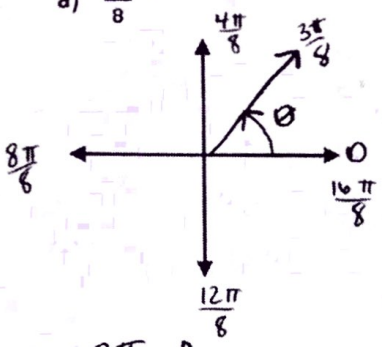
degrees is review for Math 2

Now to find reference and coterminal angles that are not using angle form that circle.

Sketch a picture of the angle θ in standard position. Then find the reference angle (2) and two coterminal angles, one positive and one negative. (3)

How many eights would it take to cover a circle? 16 What is half of eight? 4

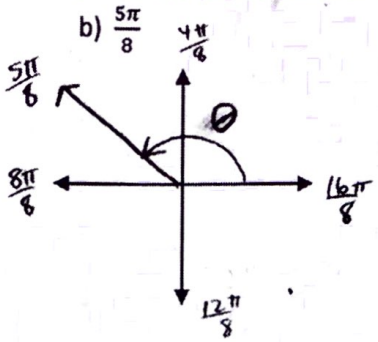
a) $\frac{3\pi}{8}$



$\frac{3\pi}{8} = \beta$

$\frac{19\pi}{8}$, $\frac{-13\pi}{8}$

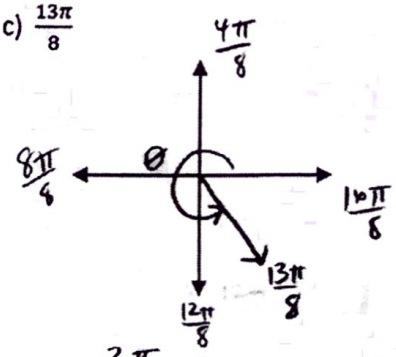
b) $\frac{5\pi}{8}$



$\frac{3\pi}{8} = \beta$

$\frac{21\pi}{8}$, $\frac{-11\pi}{8}$

c) $\frac{13\pi}{8}$

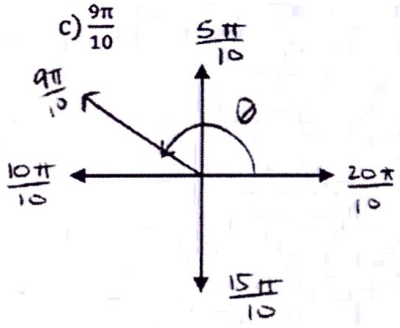


$\frac{3\pi}{8} = \beta$

$\frac{29\pi}{8}$, $\frac{-3\pi}{8}$

How many tenths would it take to cover a circle? 20 What is half of ten? 5

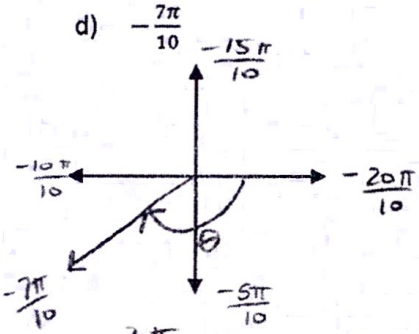
c) $\frac{9\pi}{10}$



$\beta = \frac{\pi}{10}$

$\frac{29\pi}{10}$, $\frac{-11\pi}{10}$

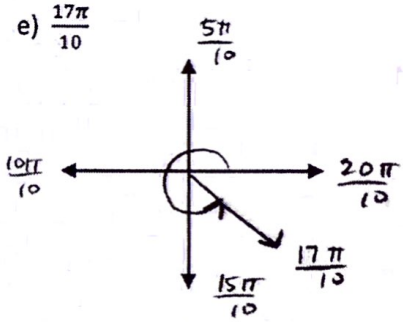
d) $-\frac{7\pi}{10}$



$\frac{3\pi}{10} = \beta$

$\frac{13\pi}{10}$, $\frac{-29\pi}{10}$

e) $\frac{17\pi}{10}$

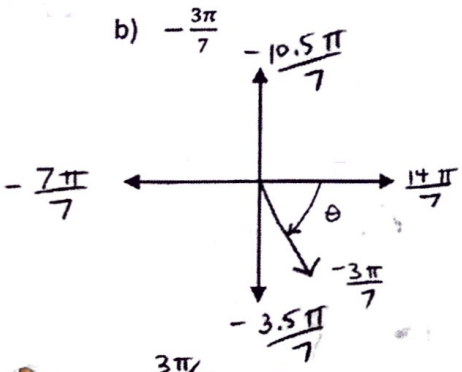


$\beta = 3\pi/10$

$\frac{37\pi}{10}$, $\frac{-3\pi}{10}$

How many sevenths would it take to cover a circle? 14 What is half of seven? 3.5

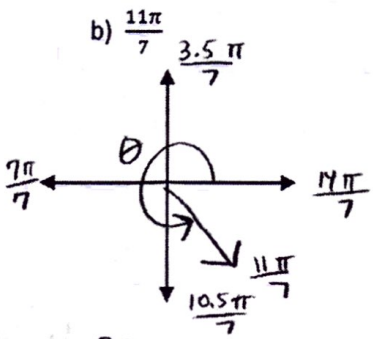
b) $-\frac{3\pi}{7}$



$3\pi/7$

$\frac{11\pi}{7}$, $\frac{-17\pi}{7}$

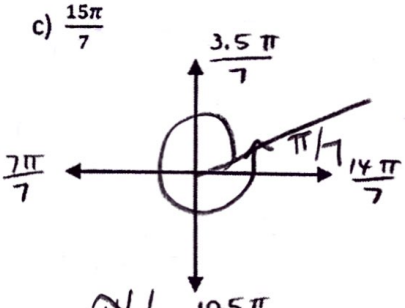
b) $\frac{11\pi}{7}$



$\frac{3\pi}{7}$

$\frac{25\pi}{7}$, $\frac{-3\pi}{7}$

c) $\frac{15\pi}{7}$



$\pi/7$

$\frac{\pi}{7}$, $\frac{-13\pi}{7}$