

10.8 NATURAL LOGARITHMS

Euler $e \approx 2.718$ $\ln / \ln / \ln$

$$\log_e x = \ln x$$

↑
understood 'e'

EXP.

LOG

$$e^2 \approx 7.39 \rightarrow \ln_e 7.39 \approx 2$$

$$e^{1.39} \approx 4 \leftarrow \ln_e 4 \approx 1.39$$

$$\begin{array}{l} \ln_e e^4 = x \\ e^x = e^4 \end{array}$$

Evaluate.

$$\textcircled{1} \ln_e e = 1$$

$$\begin{array}{l} \ln_e e = x \\ e^x = e \end{array}$$

$$\textcircled{2} \ln_e e^4 = 4$$

$$\begin{array}{l} 4 \cdot \ln_e \\ 4 \cdot 1 = 4 \end{array}$$

$$\textcircled{3} \ln_e 1 = x = 0$$
$$e^x = 1$$

$$\textcircled{4} \ln \frac{1}{e} = -1$$

$$\textcircled{5} \ln 7 - \ln 2 + \ln 8$$

$$\ln \left(\frac{7 \cdot 8}{2} \right) = \ln 28$$

$$\begin{array}{l} \ln_e e^{-1} = x \\ e^x = e^{-1} \end{array}$$

$$\textcircled{6} \quad \frac{1}{2} \ln 4 - 1$$
$$\ln 4^{\frac{1}{2}} - \ln e$$
$$\ln \left(\frac{2}{e} \right)$$

SOLVE.

$$\textcircled{1} \quad \ln_e x = 5$$
$$\boxed{e^5 = x}$$

$$\textcircled{2} \quad e^x = 3$$
$$\boxed{\ln_e 3 = x}$$

$$\textcircled{3} \quad \ln_e (x+2) = 4$$
$$e^4 = x+2$$
$$\boxed{e^4 - 2 = x}$$

$$\textcircled{4} \quad e^{x+1} = 7$$
$$\ln_e 7 = x+1$$

$$\boxed{(\ln 7) - 1 = x}$$

$$\textcircled{5} \quad e^{\ln_e 5} = x = \boxed{5}$$
$$\ln_e x = \ln_e 5$$

Natural Logarithms

Express as a single logarithm or number.

1. $2 \ln x + 3 \ln y$
 $\ln x^2 + \ln y^3$
 $\ln(x^2 y^3)$

2. $\ln c + \frac{\ln a}{3} + \frac{\ln b}{3}$
 $\ln c \sqrt[3]{ab}$

3. $\frac{1}{2} \ln 4 - \frac{1}{3} \ln 8$
 $\ln \frac{4^{1/2}}{8^{1/3}} = \ln \frac{2}{2}$
 $= \ln 1 = 0$

4. $\frac{\ln x}{2} + \frac{\ln y}{2} + \frac{\ln z}{2}$
 $\frac{1}{2} \ln x + \frac{1}{2} \ln y + \frac{1}{2} \ln z$
 $\ln \sqrt{xyz}$

5. $\ln a + \ln b + 2 \ln c$
 $\ln(abc^2)$

6. $3 \ln 3 - 4 \ln x$
 $\ln \frac{3^3}{x^4} = \ln \frac{27}{x^4}$

Solve each equation

7. $\ln x + \ln 2x = \ln e^3$
 $\ln 2x^2 = \ln e^3$
 $\frac{2x^2}{x^2} = \frac{e^3}{e^3}$
 $x = \sqrt{\frac{e^3}{2}}$

8. $\ln(4x-1) = \ln e^{36}$
 $4x-1 = e^{36}$
 $4x = e^{36} + 1$
 $x = \frac{e^{36} + 1}{4}$

9. $\ln(3x+5) = 4$
 $e^4 = 3x+5$
 $\frac{e^4 - 5}{3} = x$

10. $\ln(2x-1) = 0$
 $e^0 = 2x-1$
 $1 = 2x-1$
 $2 = 2x$
 $x = 1$

11. $\ln 3 + 2 \ln x = 0$
 $\ln 3 \cdot x^2 = 0$
 $e^0 = 3x^2$
 $1 = 3x^2$
 $\frac{1}{3} = x^2$
 $x = \sqrt{\frac{1}{3}}$
 $x = \frac{\sqrt{3}}{3}$

12. $e^{2x} = 10$
 $\ln_e 10 = 2x$
 $\frac{\ln 10}{2} = x$

13. $e^{x+1} = 30$
 $\ln_e 30 = x+1$
 $(\ln 30) - 1 = x$

14. $e^x = 18$
 $\ln_e 18 = x$

15. $e^{\frac{x}{5}} + 4 = 7$
 $e^{\frac{x}{5}} = 3$
 $\ln_e 3 = \frac{x}{5}$
 $5 \ln 3 = x$

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