CHANGE OF BASE FORMULA / NATURAL LOGARITHMS

$$\log_3 7 = \frac{\log 7}{\log 3} \approx 1.771$$

Change of base: $\log_a n = \frac{\log_{10} n}{\log_{10} a}$

Solve.

① $\log 9^b = \log 45$ —— $\log_9 45 = b$

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$\frac{b \cdot \log 9}{\log 9} = \frac{\log 45}{\log 9}$

$b = 1.732$

② $\log 3.1^{a-3} = \log 9.42$

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$(a-3)\log 3.1 = \log 9.42$

$\frac{(a-3)\log 3.1}{\log 3.1} = \frac{\log 9.42}{\log 3.1}$

$a-3 = 1.982$

$+3 \quad +3$

$\boxed{a = 4.982}$

LN / ln / ln

$P(1+\frac{r}{n})^{nt}$

Euler number $e \approx 2.718$

$\lim_{x \to \infty} 1(1+x)^x$

$\boxed{\log_e x \to \ln x}$

$\ln_e e^1 = 1$

Solve.

① $\ln_e \frac{1}{e} = \ln_e e^{-1} = -1 \cdot \ln_e e = \boxed{-1}$

$\log_2 \sqrt{e}$

② $\ln \sqrt{e} = \ln_e e^{\frac{1}{2}} = \boxed{\frac{1}{2}}$

③ $\ln 7 - \ln 2 + \ln 8 =$

$\ln 7 + \ln 8 - \ln 2 = \ln \frac{7 \cdot 8}{2}$

$= \ln \frac{56}{2} = \boxed{\ln 28}$

④ $\ln_e (x-4) = -1$ (Leave in terms of e and ln!)

$e^{-1} = x - 4$

$\frac{1}{e} = x - 4$

$+4 \quad +4$

$\boxed{\frac{1}{e} + 4 = x}$

⑤ mult.

$\ln x + \ln (x+3) = \ln 10$

$\ln (x^2 + 3x) = \ln 10$

$x^2 + 3x = 10$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = \cancel{-5}, 2$

⑥ $e^{-x} = 3$

$\frac{\ln_e 3}{-1} = \frac{-x}{-1}$

$-\ln 3 = x$

$\ln 3^{-1} = \ln \frac{1}{3}$

isolate e^{2x}

⑦ $3e^{2x} + 2 = 50$

$\frac{3e^{2x}}{3} = \frac{48}{3}$

$e^{2x} = 16$

$\frac{\ln_e 16}{2} = \frac{2x}{2}$

$\frac{\ln 16}{2} = x$

$\frac{1}{2}\ln 16 = x = \ln 4$

⑧ $e^{x+3} + 2 = 8$

$\ln(6)^{-3} = x$

⑨ $\ln\left(\frac{1}{x}\right) = 4$

$x = \frac{1}{e^4}$