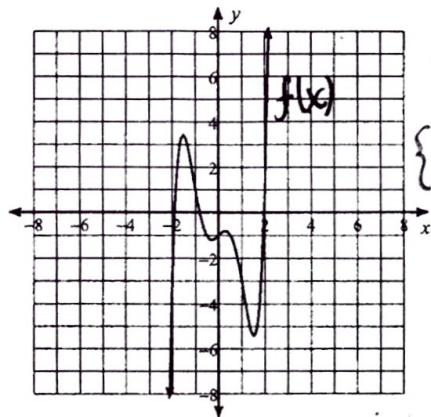


Polynomial Inequalities

For what interval is  $f(x) < 0$ ? For what interval is  $f(x) \geq 0$ ?

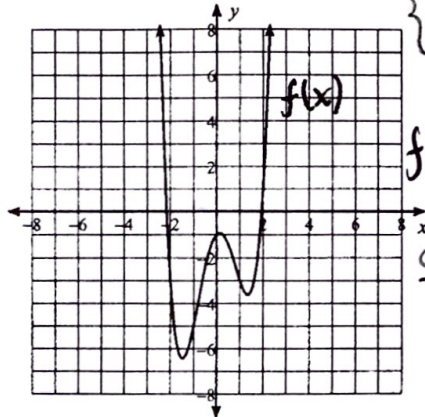
1)  $< 0 \{x | x < -2 \text{ or } -1 < x < 2\}$



$\geq 0 \{x | -2 \leq x \leq -1, x \geq 2\}$

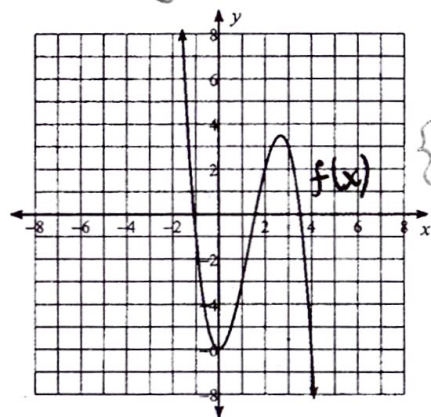
2)

$f(x) < 0 \{x | -2 < x < 2\}$



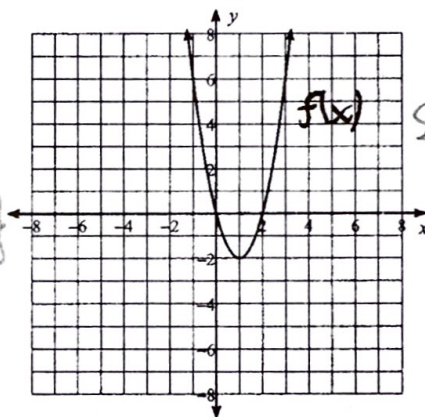
$f(x) \geq 0 \{x | x \leq -2 \text{ or } x \geq 2\}$

3)  $< 0 \{x | -1 < x < 1.5 \text{ or } x > 3.5\}$



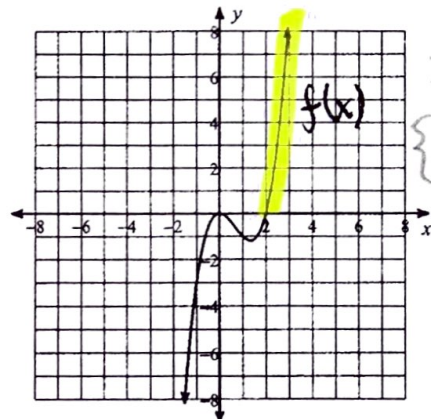
$\geq 0 \{x | x \leq -1 \text{ or } 1.5 \leq x \leq 3.5\}$

4)  $< 0: \{x | 0 < x < 2\}$



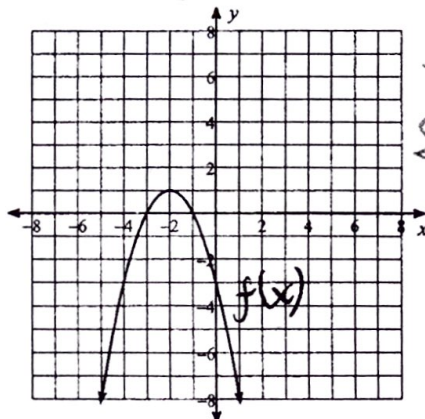
$\geq 0 \{x | x \leq 0 \text{ or } x \geq 2\}$

5)  $< 0 \{x | x < 2, x \neq 0\}$



$\geq 0 \{x | x \geq 2\} \text{ or } x = 0$

6)  $< 0 \{x | x < -3 \text{ or } x > -1\}$



$\geq 0 \{x | -3 \leq x \leq -1\}$

# INEQUALITY → CLASS EXAMPLES

Find the roots. Identify any multiple roots.

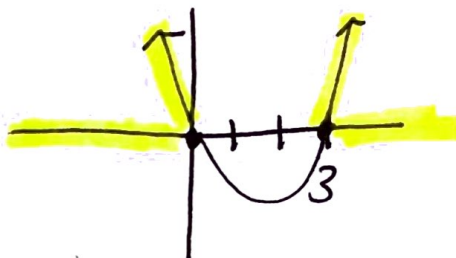
1.  $y^2 - 3y = 0$

$$y^2 - 3y \geq 0$$

find roots

$$y(y-3) \geq 0$$

$y = 0, 3$  - sketch graph



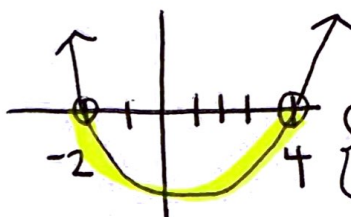
$$\{y \mid y \leq 0 \text{ or } y \geq 3\}$$

2.  $r^2 - 2r - 8 = 0$

$$r^2 - 2r - 8 < 0$$

$$(r+2)(r-4) < 0$$

roots:  $r = -2, 4$  → sketch graph

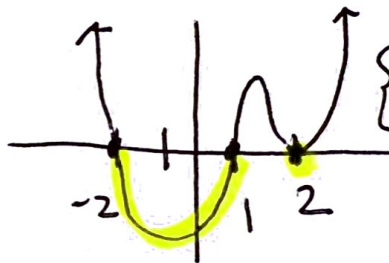


$$\{r \mid -2 < r < 4\}$$

3.  $(x^2 + x + 2)(x^2 - 4x + 4) = 0$

$$(x-1)(x+2)(x-2)(x-2) \leq 0$$

roots:  $1, -2, 2$  d.r.



$$\{x \mid -2 \leq x \leq 1 \text{ or } x = 2\}$$

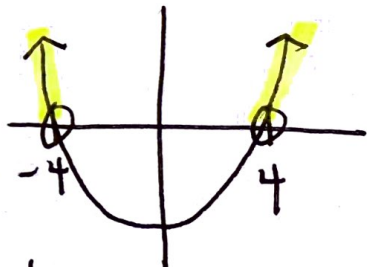
4.  $3x^2 = 48$

$$3x^2 > 48$$

$$3x^2 - 48 > 0$$

$$3(x^2 - 16) = 3(x+4)(x-4) > 0$$

roots  $x = \pm 4$



$$\{x \mid x < -4 \text{ or } x > 4\}$$

5.  $(t-2)^3 - (t-2) = 0$

$$(t-2)^3 - (t-2) \geq 0$$

$$a^3 - a \geq 0$$

$$a(a^2 - 1) \geq 0 \rightarrow (t-2)(t-1)(t-3) \geq 0$$

$$a(a+1)(a-1)$$

$t = 2, 1, 3$

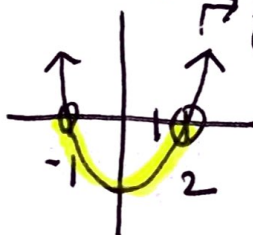
6.  $x+1 = (x+1)(x-1)$

$$x+1 > (x+1)(x-1) \quad x = -1, 2$$

$$x+1 > x^2 - 1$$

$$0 > x^2 - x - 2$$

$$x^2 - x - 2 < 0$$



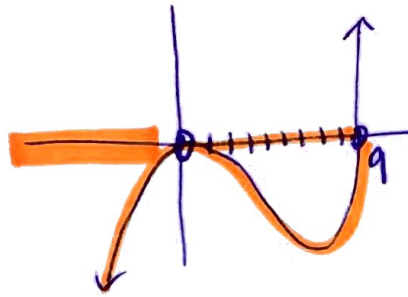
$$\{x \mid -1 < x < 2\}$$

$$\{t \mid 1 \leq t \leq 2 \text{ or } t \geq 3\}$$

Solve.

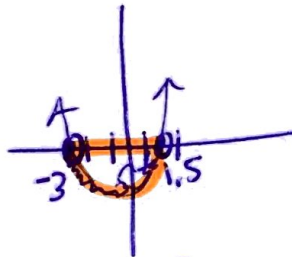
# Math Key

1.  $t^3 < 9t^2$   
 $t^3 < 9t^2$   
 $t^3 - 9t^2 < 0$   
 $t^2(t-9) < 0$   
 $t=0$  OR  
 $t=9$



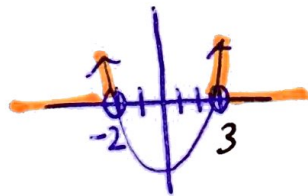
$\{t \mid t < 0, t \neq 0\}$   
 OR  $\{t \mid 0 < t < 9\}$

2.  $9 - 3t - 2t^2 > 0$   
 $-2t^2 - 3t + 9 > 0$   
 $-(2t^2 + 3t - 9) > 0$   
 $2t^2 + 3t - 9 < 0$   
 $(2t-3)(t+3) < 0$   
 $t = \frac{3}{2}$   
 $t = -3$



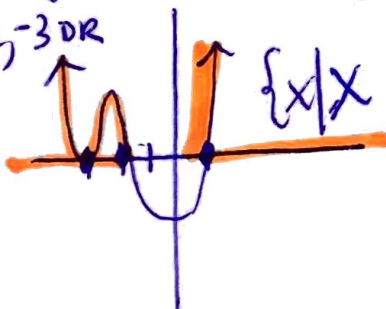
$\{t \mid -3 < t < 1.5\}$

3.  $(x^2 - 1) > (x + 5)$   
 $x^2 - x - 6 > 0$   
 $(x+2)(x-3) > 0$   
 $x = -2$   
 $x = 3$



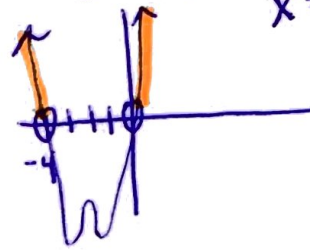
$\{x \mid x < -2 \text{ or } x > 3\}$

4.  $(x^2 + 2x - 3)(x^2 + 5x + 6) \geq 0$   
 $(x-1)(x+3)(x+2)(x+3) \geq 0$   
 $x=1, -2, -3$  OR



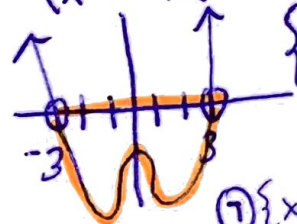
$\{x \mid x \leq -2 \text{ or } x \geq 1\}$

5.  $(x^2 + 4)(x^2 + 4x) > 0$   
 $(x^2 + 4)(x(x+4)) > 0$   
 $x = 0, -4, \pm 2i$

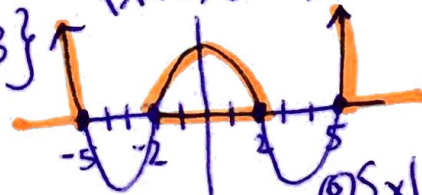


⑤  $\{x \mid x < -4 \text{ or } x > 0\}$

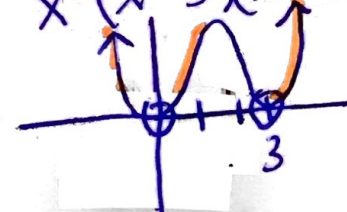
6.  $x^4 - 18 < 7x^2$   
 $x^4 - 7x^2 - 18 < 0$   
 $(x^2 + 2)(x^2 - 9) < 0$   $x = \pm 3, \pm i\sqrt{2}$   
 $(x^2 + 2)(x+3)(x-3) < 0$   
 $\{x \mid -3 < x < 3\}$



7.  $x^4 + 100 \geq 29x^2$   
 $x^4 - 29x^2 + 100 \geq 0$   
 $(x^2 - 4)(x^2 - 25) \geq 0$   
 $(x+2)(x-2)(x+5)(x-5) \geq 0$   
 $x = \pm 2, \pm 5$   
 ⑦  $\{x \mid x \leq -5 \text{ or } -2 \leq x \leq 2 \text{ or } x \geq 5\}$



8.  $x^2(x^2 + 9) > 6x^3$   
 $x^4 + 9x^2 - 6x^3 > 0$   
 $x^2(x^2 - 6x + 9) > 0$   
 $x^2(x-3)(x-3) > 0$   
 $x = 0$  OR  $3$   
 ⑧  $\{x \mid x \neq 0, x \neq 3\}$



$x = 0$  OR  $3$