A **prism** is a polyhedron with two parallel, congruent faces called **bases**. A **pyramid** is a polyhedron with one base that is a polygon and faces that are triangles. Prisms and pyramids are named by the shape of their bases.

There are also solids that are not polyhedrons. A **cylinder** is a three-dimensional figure with congruent, parallel bases that are circles connected with a curved side. A **cone** has one circular base and a vertex connected by a curved side.

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**Study Tip**

*Common Error* In the drawing of a rectangular prism, the bases do not have to be on the top and bottom. Any two parallel rectangles are bases. In a triangular pyramid, any face is a base.

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**EXAMPLES**

### Identify Solids

Identify the figure. Then name the bases, faces, edges, and vertices.

1. The figure has two parallel congruent bases that are triangles, so it is a triangular prism.
   - **bases** $ABE$, $FCD$
   - **faces** $ABE$, $FCD$, $BCDE$, $FAED$, $ABCF$
   - **edges** $AB$, $BE$, $EA$, $FC$, $CD$, $DF$, $BC$, $ED$, $AF$
   - **vertices** $A$, $B$, $C$, $D$, $E$, $F$

2. The figure has one base that is a pentagon, so it is a pentagonal pyramid.
   - **base** $RSTUV$
   - **faces** $RSTUV$, $QVR$, $QRS$, $QST$, $QTU$, $QUV$
   - **edges** $QR$, $QS$, $QT$, $QU$, $QV$, $VR$, $RS$, $ST$, $TU$, $UV$
   - **vertices** $Q$, $R$, $S$, $T$, $U$, $V$

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**CHECK Your Progress**

- **a.**
- **b.**
- **c.**
You can use three-dimensional drawings of objects to describe how different parts of the objects are related in space.

**REAL-WORLD EXAMPLE** Analyze Drawings

For Furniture: The photo shows a garden bench. Draw and label the top, front, and side views of the bench.

<table>
<thead>
<tr>
<th>Top</th>
<th>Front</th>
<th>Side</th>
</tr>
</thead>
</table>

**CHECK Your Progress**

d. Toolbox: Draw and label the top, front, and side views of the toolbox shown.

The intersection of a solid and a plane is called a **cross section** of the solid.

**EXAMPLE** Identify Cross Sections

Describe the shape resulting from a vertical, angled, and horizontal cross section of a cylinder.

- Vertical Slice
- Angled Slice
- Horizontal Slice

The cross section is a rectangle. The cross section is an oval. The cross section is a circle.

**CHECK Your Progress**
e. Describe the shape resulting from a vertical, angled, and horizontal cross section of a square pyramid.
Check Your Understanding

Examples 1 and 2  Identify each figure. Then name the bases, faces, edges, and vertices.

1. Rectangular prism

2. Triangular prism

3. Cylinder

Example 3  4. Aquariums Draw and label the top, front, and side views of the aquarium shown.

Example 4  Describe the shape resulting from each cross section.

5. Circle

6. Triangle

7. Rectangle

Practice and Problem Solving

Examples 1 and 2  Identify each figure. Then name the bases, faces, edges and vertices.

8.

9.

10.

Example 3  11. Buildings Draw and label the top, front, and side views of the building.

12. Tent Draw and label the top, front, and side views of the tent.
Example 4 Describe the shape resulting from each cross section.

13. rectangle
14. triangle
15. triangle
16. square
17. semicircle
18. triangle

19. State whether the following conjecture is true or false. If false, provide a counterexample.

Two planes in three-dimensional space can intersect at one point.

20. SPORTS A standard basketball is shaped like a sphere.
   a. Draw a basketball with a vertical, angled, and horizontal slice.
   b. Describe the cross section made by each slice.
      All circles

H.O.T. Problems

21. OPEN ENDED Draw the cross sections of a polyhedron, cylinder, or cone. Exchange papers with another student. Identify the three-dimensional figures represented by the cross sections.

22. FIND THE ERROR Brian is identifying the figure below. Find his mistake and correct it.

The figure has a triangular base. It is a triangular pyramid.

CHALLENGE Determine whether each statement is always, sometimes, or never true. Explain your reasoning.


25. WRITE MATH Explain whether a top-front-side view diagram always provides enough information to draw a figure. If not, provide a counterexample.
12-1 Study Guide and Intervention (continued)

Representations of Three-Dimensional Figures

Cross Sections The intersection of a solid and a plane is called a cross section of the solid. The shape of a cross section depends upon the angle of the plane.

Example

There are several interesting shapes that are cross sections of a cone. Determine the shape resulting from each cross section of the cone.

a. If the plane is parallel to the base of the cone, then the resulting cross section will be a circle.

b. If the plane cuts through the cone perpendicular to the base and through the center of the cone, then the resulting cross section will be a triangle.

c. If the plane cuts across the entire cone, then the resulting cross section will be an ellipse.

Exercises

Describe each cross section.

1. Circle
2. Oval / Ellipse
3. Rectangle
26. Benita received the gift box shown.

Which drawing best represents the top view of the gift box?

A. 

B. 

C. 

D. 

27. Which of the following is NOT an example of a polyhedron?
   - F. cylinder
   - G. rectangular prism
   - H. octagonal pyramid
   - I. triangular prism

28. Which of the following represents a side view of the figure below?

A. 

C. 

D. 

29. The figure below is a square pyramid.

Which of the following is NOT a cross section from the square pyramid?

F. 

H. 

G. 

I.
Cross sections of 3D objects (basic)

You're out on a camping trip, and you're thinking about the geometry of your triangular prism.

Which shape does the intersection of the vertical plane with the tent look like?

Choose 1 answer:

A

B

C

D

Cross sections of 3D objects (basic)

A vertical slice through a three-dimensional solid produces a two-dimensional shape.

Which one of the following solids can produce this two-dimensional shape?

Choose 1 answer:

A

B

C

D
A horizontal slice through a three-dimensional solid produces a two-dimensional shape.

Which one of the following solids produces this two-dimensional shape?

Choose 1 answer:

Which shape does the intersection of the horizontal plane with the prism?

Choose 1 answer:

The figure below shows a right regular prism whose base is a pentagon.
2D Rotations to 3D Shapes

1. Describe in detail the solid formed by rotating a right triangle with vertices at (0, 0), (2, 0), and (0, 3) about the vertical axis. Include the dimensions (height, length, width, radius, etc) of the solid in your description.

Cone pointing up
h = 3 units
radius = 2 units

2. Describe in detail the solid formed by rotating a right triangle with vertices at (0, 0), (2, 0), and (0, 3) about the horizontal axis. Include the dimensions (height, length, width, radius, etc) of the solid in your description.

Cone pointing right
h = 2 units
r = 3 units

3. Imagine the solid formed by rotating the same right triangle about the line x = 2. Describe this solid in detail including its dimensions.

Cylinder with cone removed
h = 3 units
r = 2 units

4. Describe in detail the solid formed by rotating a 2 x 3 rectangle with vertices (2, 0), (4, 0), (2, 3) and (4, 3) about the x-axis. Include the dimensions (height, length, width, radius, etc) of the solid in your description.

Cylinder
h = 2 units
r = 3 units

5. Describe in detail the solid formed by rotating a 2 x 3 rectangle with vertices (2, 0), (4, 0), (2, 3) and (4, 3) about the y-axis. Include the dimensions (height, length, width, radius, etc) of the solid in your description.

Big cylinder with small cylinder removed
h = 3
r = 4
Go to the following website: CK-12 SLT3 Identity the solid created by rotating a two-dimensional figure
https://www.ck12.org/geometry/cross-sections-and-basic-solids-of-revolution/lesson/user:ZGVib3JhaF9hX2hpbHRuZXJAbWNwc21kLm9yZw./SLT-3-Identify-the-solid-created-by-rotating-a-two-dimensional-figure/

Click on the link under “Watch this”

You should be at Predict 3D results of rotating simple figures. Under Direction Instruction, watch the video.

1. What is a rotation?  
   circular movement around a center of rotation

2. What is the axis of rotation?  
   imaginary line of rotation for 3D objects

First Triangle:
3. a) What did the edges perpendicular to the axis draw?  
   flat surfaces

   b) What did the edges parallel to the axis draw?  
   curved surfaces

   c) What 3D shape was created from rotating the triangle?  
   cone

First rectangle:
4. a) What did the edges perpendicular to the axis draw?  
   flat surfaces

   b) What did the edges parallel to the axis draw?  
   curved surfaces

   c) What 3D shape resulted from rotating the rectangle?  
   cylinder

Second rectangle: diagonal
5. a) What did the edges perpendicular to the axis draw?  
   flat faces

   b) What did the edges parallel to the axis draw?  
   curved surfaces
c) What 3D shape resulted from rotating the rectangle? **cylinder**

Third rectangle:

6. What 3D shape resulted from rotating the rectangle this time? **diamond on top of diamond**

First circle:

7. a) What did the curved edges draw? **curved surface**

b) What 3D shape resulted from rotating the rectangle? **sphere**

Once you are done with this video, you have completed this task!