

8.5 - Remainder Theorem & Factor Theorem

just #1 ≠ TALK!

1. a) Divide using synthetic division $(2x^2 - 4x + 3) \div (x - 3)$

$$\begin{array}{r|rrr} 3 & 2 & -4 & 3 \\ & & 6 & 6 \\ \hline & 2 & 2 & 9 \end{array}$$

$$\boxed{2x + 2 + \frac{9}{x-3}}$$

b) Find $f(3)$ if $f(x) = 2x^2 - 4x + 3$ = **SYNTHETIC SUBSTITUTION!** -on HW 1,5

$$f(3) = 2(3)^2 - 4(3) + 3$$

$$= 2(9) - 12 + 3 = 18 - 12 + 3 = 9$$

c) What do you notice?

$f(3)$ = remainder when dividing by $(x-3)$

2. a) Is 4 a root of $f(x) = x^4 - 6x^3 + 8x^2 + 5x + 3$? **NO**

$$\begin{array}{r|rrrrr} 4 & 1 & -6 & 8 & 5 & 3 \\ & & 4 & -8 & 0 & 20 \\ \hline & 1 & -2 & 0 & 5 & 23 \end{array}$$

b) Is $x - 4$ a factor of $x^4 - 6x^3 + 8x^2 + 5x + 3$?

NO

c) How do you know? remainder $\neq 0$

3. Is $(x + 1)$ a factor of $P(x) = x^{100} + 2x^3 + 1$? Explain your answer.

$x+1=0$
 $x=-1$

$$P(-1) = (-1)^{100} + 2(-1)^3 + 1$$

$$= 1 - 2 + 1 = 0$$

FACTOR THEOREM 9,13
yes, $P(-1) = 0!$

Try to Factor 1st!

(like p.375 #17,19)

1. Solve $2x^3 - 5x^2 - 4x + 3$, given that 3 is a root.

$$\begin{array}{r|rrrr} 3 & 2 & -5 & -4 & 3 \\ & & 6 & 3 & -3 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$\boxed{x = \frac{1}{2}, -1, 3}$$

2. Solve $2m^3 - 5m^2 - 13m - 5$, given that $-1/2$ is a root.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -5 & -13 & -5 \\ & & -1 & 3 & 5 \\ \hline & 2 & -6 & -10 & 0 \end{array}$$

$$2m^2 - 6m - 10 = 2(m^2 - 3m - 5)$$

$$m = \frac{3 \pm \sqrt{9 - 4(1)(-5)}}{2} = \frac{3 \pm \sqrt{29}}{2}, -\frac{1}{2}$$

3. Find a polynomial equation with integral coefficients that has the given numbers as roots.

a) 3 (double), 1, -2

b) 5, $1+3i$, $1-3i$

S: 6
P: 9

$$(x^2 - 6x + 9)(x^2 + x - 2)$$

$$x^4 + x^3 - 2x^2 + 12x - 18$$

$$-6x^3 - 6x^2 + 9x$$

$$+ 9x^2$$

$$\boxed{x^4 - 5x^3 + x^2 + 21x - 18}$$

$$(x-5)(x^2 - 2x + 10)$$

$$x^3 - 2x^2 + 10x - 50$$

$$-5x^2 + 10x$$

$$\boxed{x^3 - 7x^2 + 20x - 50}$$

4. Solve each equation given the two indicated roots.

$2x^4 - 5x^3 - 11x^2 + 20x + 12$; -2, 3

like 29,31

Could make into quadratic...
do long!

$$\begin{array}{r|rrrrr} -2 & 2 & -5 & -11 & 20 & 12 \\ & & -4 & 18 & -14 & -12 \\ \hline & 2 & -9 & 7 & 6 & 0 \\ & & 6 & -9 & -6 & \\ \hline & 2 & -3 & -2 & 0 & \end{array}$$

$$2x^2 - 3x - 2 = (2x+1)(x-2)$$

$$\boxed{x = -2, 3, -\frac{1}{2}, 2}$$

5. Find the number of times 2 is a root of $x^4 - 5x^3 + 6x^2 + 4x - 8$.

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 6 & 4 & -8 \\ & & 2 & -6 & 0 & 8 \\ \hline & 1 & -3 & 0 & 4 & 0 \\ & & 2 & -2 & -4 & \\ \hline & 1 & -1 & -2 & 0 & \\ & & 2 & 2 & \\ \hline & 1 & 1 & 0 & \end{array}$$

3 times