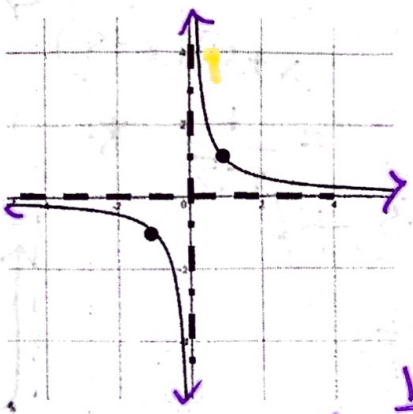
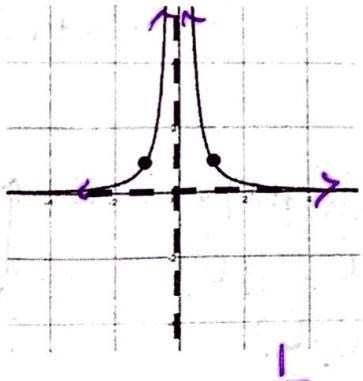
Graphing Rational Functions



Parent function $y = \frac{1}{x}$

Horizontal asymptote y = 0/ Vertical asymptote at x = 0



Parent function $y = \frac{1}{x^2}$

Horizontal asymptote at y = 0/ Vertical asymptote at x = 0

Ex. 1 $f(x) = \frac{2x}{x+1}$

zeros → $2x = 0$

VA → $x+1 = 0$

$x = -1$

x	y
-3	$\frac{-6}{-2} = 3$

Ex. 2 $f(x) = \frac{2}{(x+1)^2} = (x+1)(x+1)$

zeros → $x+1 = 0$

$x = -1$

Domain: $\{x \mid x \neq -1\}$

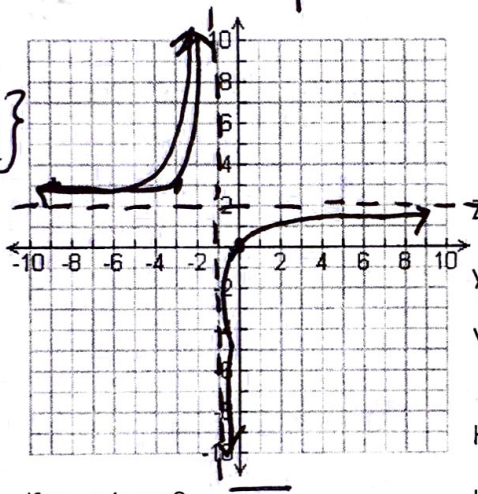
Zeros: $(0,0)$

y-intercept: $(0,0)$

VA $x = -1$

HA $y = 2$

Is there a hole? no If so, where? _____



Domain: $\{x \mid x \neq -1\}$

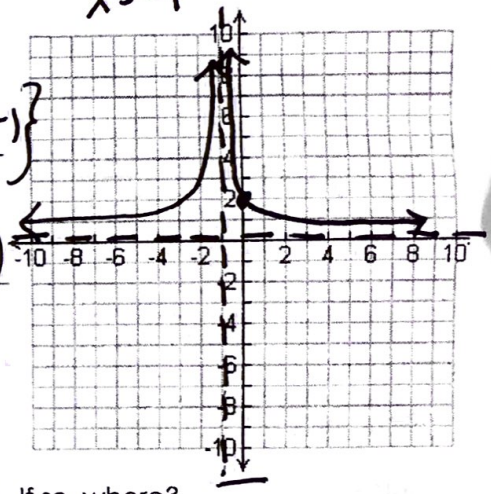
Zeros: none

y-intercept: $(0,2)$

VA $x = -1$

HA $y = 0$

Is there a hole? no If so, where? _____



Vertical asymptote (VA): set denom. = 0 ($x = \#$)

Horizontal asymptote (HA): degree of num. > degree of denom. → HA: none

degree of num. = degree of denom. → HA: $y = \#$ (use coefficients!)

degree of num. < degree of denom. → HA: $y = 0$

Zeros: set numerator = 0

y-int.: sub. $x = 0$

Domain: rule out VA value / hole x-value

Hole: if matching factor num./denom.

Ex. 3 $f(x) = \frac{x^2-x-6}{x^2-1} = \frac{(x+2)(x-3)}{(x+1)(x-1)}$

Domain: $\{x \mid x \neq \pm 1\}$

Zeros: $(-2,0)\ (3,0)$

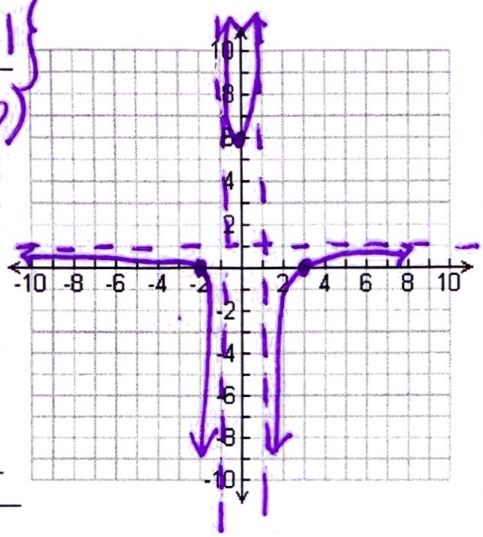
y-intercept: $(0,6)$

VA $x=1, x=-1$

HA $y=1$

Is there a hole? no

If so, where? —



Ex. 4 $f(x) = \frac{x+5}{x^2-25} = \frac{x+5}{(x+5)(x-5)} = \frac{1}{x-5}$

→ hole o

Domain: $\{x \mid x \neq \pm 5\}$

Zeros: none

y-intercept: $(0, -\frac{1}{5})$

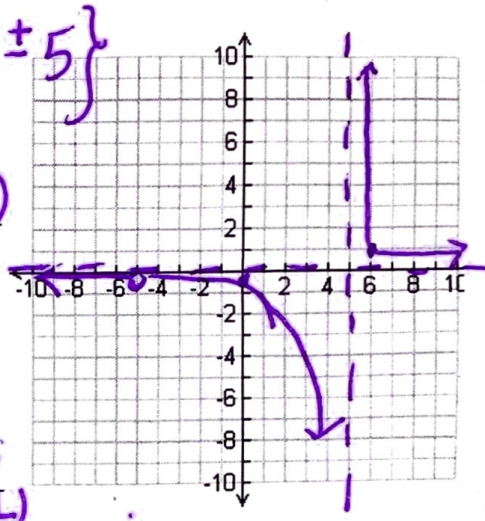
VA $x=5$

HA $y=0$

Is there a hole? yes

If so, where? $(-5, -\frac{1}{10})$

$\frac{1}{-5-5}$



Ex. 5 $f(x) = \frac{x+2}{x^2+x-12} = \frac{x+2}{(x-3)(x+4)}$

Domain: $\{x \mid x \neq 3, -4\}$

Zeros: $(-2,0)$

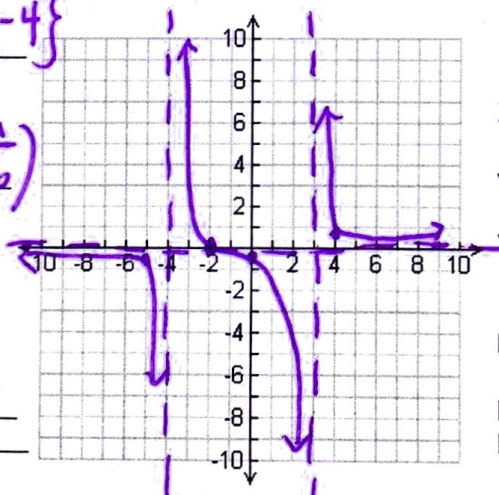
y-intercept: $(0, -\frac{1}{6})$

VA $x=3, -4$

HA $y=0$

Is there a hole? no

If so, where? —



$\frac{-5+2}{25-5-12} = \frac{-3}{8}$ $(-5, -\frac{3}{8})$

$\frac{4+2}{16+4-12} = \frac{6}{8} = (4, \frac{3}{4})$

Ex. 6 $f(x) = \frac{x^2-3x+2}{x^2+x-2}$

Domain: _____

Zeros: _____

y-intercept: _____

VA _____

HA _____

Is there a hole? _____

If so, where? _____