NATURAL LOG → $\ln x = \log_e x$ understood base 'e'

$\boxed{\text{P.40}}$

Solve

① $\frac{4e^x}{4} = \frac{20}{4}$

rewrite in log form: $e^x = 5$, $\ln_e 5 = \boxed{x = 1.609}$

② $\ln_e x = 7$

$e^7 = x$

$\boxed{1096.633 = x}$

Math 3

Unit 3 Exponential & Logarithmic Functions Name

Homework ~ Natural Logarithms and Base e.

Evaluate each of the following. Round your answers to three decimal places.

1. $2e^4$ 109.196

2. $-3+e^2$ 4.389

3. $\ln 2.9$ 1.065

4. $8+\ln 16$ 10.773

5. $3\ln 12 + \ln 4$ 8.841

6. $e^{\ln 2}$ 2

Expand each of the following using the properties of logarithms.

7. $\ln 5x^3$

8. $\ln \frac{w}{8}$

9. $\ln 12kj$

10. $\ln 6x^4y^3z^2$

11. $\ln \frac{ma}{b}$

12. $\ln \frac{x}{ab^5}$

Use the properties of logarithms to condense each of the following.

13. $2\ln a + \ln 3 - \frac{1}{3}\ln c$

14. $\ln h - \ln 4$

15. $\frac{1}{2}\ln w - (\ln x + 4\ln h)$

16. $\ln x - 3\ln f$

17. $15\ln a - 6\ln b$

18. $4\ln x + \ln 7 - 2\ln a$

$P = 1500$ $r = .035$ ($\frac{3.5}{100}$) $A = P(1+\frac{r}{n})^{nt}$ $A = ?$ $n = 12$

19. You deposit \$1500 into an account that pays 3.5% interest, *compounded monthly*. Find the balance in the account after 8 years.

$t = 8$ $A = 1500\left(1+\frac{.035}{12}\right)^{12\cdot 8} = \boxed{\$1983.89}$

$A = ?$

20. You deposit \$1500 into an account that pays 3.5% interest, *compounded continuously*. Find the balance in the account after 8 years.

$A = Pe^{rt} = 1500e^{(.035\cdot 8)}$

$= \$1984.69$

P.41

$A = Pe^{rt}$

$P = 400$ $r = .02$ $t = ?$

21. You deposit \$400 into an account that pays 2% interest, *compounded continuously*. How long does it take for the balance to reach \$650?

$A = 650$

$\frac{650}{400} = \frac{\cancel{400}e^{.02t}}{\cancel{400}}$

$e^{.02t} = 1.625$

$\ln_e 1.625 = .02t$

$\frac{.4855}{.02} = \frac{\cancel{.02}t}{\cancel{.02}}$

$\boxed{t = 24.3 \text{ years}}$

22. You deposit \$1000 into an account that pays 4.2% interest, *compounded continuously*. How long does it take for the balance to double?

$\frac{2000}{1000} = \frac{1000e^{.042t}}{1000}$

$e^{.042t} = 2 \leftarrow 2 = e^{.042t}$

$\ln_e 2 = .042t$

$\frac{.693}{.042} = \frac{\cancel{.042}t}{\cancel{.042}}$

$\boxed{t = 16.5 \text{ yrs.}}$

23. The half-life of copper is 60 hours. How long does it take 55 grams of iodine to decay to 8 grams?

24. The half-life of a radio active substance is 3.8 days. How long does it take 1200 grams of this substance to decay to 80 grams?

P.42

Compound Interest $A = P(1+\frac{r}{n})^{nt}$

- P = initial amount (amount deposited)
- t = time (years)
- r = rate (%) → 6% → .06 | 3.5% → .035
- n = number of times compounded each year
- A = amount in account at end

quarterly → n=4
monthly → n=12
annually → n=1
semiannually → n=2

Compound Continuously Interest $A = P_0e^{rt}$

↑ 'e' in calc

- P = initial amount (amount deposited)
- r = rate (%)
- t = time (years)
- A = amount in account at end

Appreciation/Depreciation $V = P(1 \pm r)^t$

- P = starting amount
- r = rate of appreciation (+)/depreciation (-)
- t = time
- V = value at end

Doubling Time $A = A_0(2)^{\frac{t}{d}}$

*

- A_o = initial amount
- t = time
- d = doubling time
- A = amount at end

Half Time $A = A_0(0.5)^{\frac{t}{h}}$

- A_o = initial amount
- t = time
- h = halving time
- A = amount at end

P.44 $P=6500$ $V=P(1-r)^t$ $r=.143$

2) A computer valued at $6500 depreciates at the rate of 14.3% per year. (a) Write a function that models the value of the computer. (b) Find the value of the computer after three years. $\rightarrow t=3$

model = formula!

a) $V=6500(1-.143)^t$

b) $V=6500(1-.143)^3 = \$4091.25$ $\frac{3.5}{100}$

3) The population of a certain animal species decreases at a rate of 3.5% per year. You have counted 80 of the animals in the habitat you are studying. (a) Write a function that models the change in the animal population. (b) Estimate the number of years until the population first drops below 15 animals.

$t=?$

a) $V=80(1-.035)^t$

b) $15=80(.965)^t$ $\boxed{t \rightarrow 47\text{years.}}$

CALC – TABLE

4) The value of an industrial machine decreases 25% per year. After six years, the machine is worth $7500. What was the original value of the machine?

$V=7500$

~~MODEL~~:

$7500=P(1-.25)^6$

$7500=P(.75)^6$

$\frac{7500}{.17798}=\frac{P(.17798)}{.17798}$

$P=\$42,139,92$

5) The function $y = 20(0.975)^x$ models the intensity of sunlight beneath the surface of the ocean. The output y represents the percent of surface sunlight intensity that reaches a depth of x feet. The model is accurate from about 20 feet to about 600 feet beneath the surface. (a) Find the percent of sunlight 50 feet beneath the surface of the ocean. (b) Find the percent of sunlight at a depth of 370 ft.

Write an exponential function $y = ab^x$ for a graph that includes the given points.

6) (4, 8), (6, 32)

7) (-3, 24), (-2, 12)

Exponential Growth and Decay Word Problems

1. Find a bank account balance if the account starts with \$100, has an annual rate of 4%, and the money left in the account for 12 years.

2. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

$A = A_0(2)^{\frac{t}{d}}$

3. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

$t = 24$ $A_0 = 1$ $d = 1$ $A = ?$

$A = 1(2)^{\frac{24}{1}} = \boxed{16{,}777{,}216 \text{ bacteria}}$

4. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

5. The population of Winnemucca, Nevada, can be modeled by $P=6191(1.04)^t$ where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

6. You have inherited land that was purchased for \$30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

7. During normal breathing, about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?