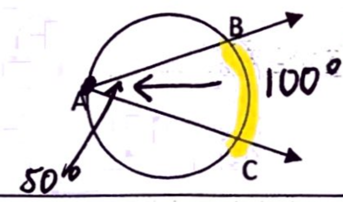
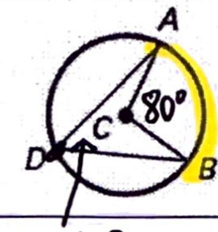
Unit 6a Day 3: More Angles of a Circle

Inscribed angle:
an angle whose vertex is on a circle and whose sides contain chords of the circle

$\angle BAC$ intercepts $\overset{\frown}{BC}$

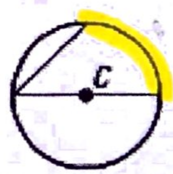


Measure of an Inscribed Angle:
the measure of an inscribed angle is one half the measure of its intercepted arc



80° central angles = intercepted arcs

EXAMPLE 3: Trace the intercepted arc of each of the inscribed angles.

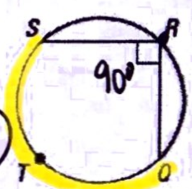
a. 

b. 

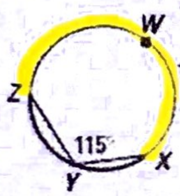
c. 

inscribed 2 · angle = arc / $\frac{1}{2}$ arc = inscribed angle

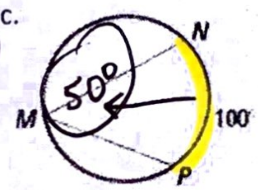
EXAMPLE 4: Finding the measure of each arc and inscribed angle.

a. 

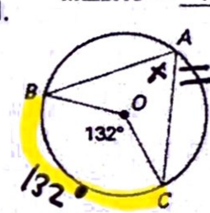
90 x 2
180°

b. 

2(115)
230°

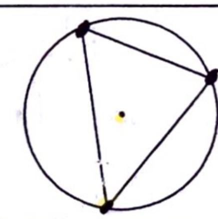
c. 

d. $m\angle BAC =$ 66°



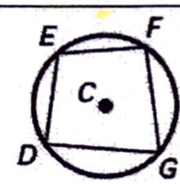
$\frac{1}{2}(132)$
66°

Inscribed Polygons:
A polygon placed inside a circle such that all the vertices of the polygon lie on the on the circle.



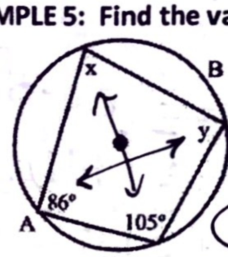
Quadrilateral RULE:
A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

→ opp. angles add to 180

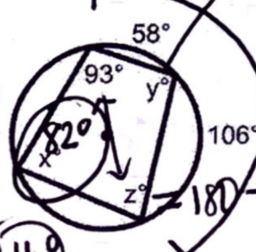


E opp. G
F opp. D

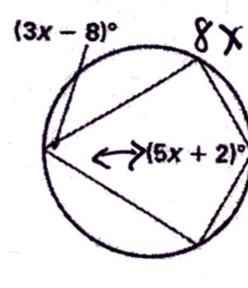
EXAMPLE 5: Find the value of each variable.

a. 

x = 180 - 105
x = 75°
y = 180 - 86 = 94°

b. 

180 - 82 = y
98 = y
164
180 - 93 = 87°

c. 

3x - 8 + 5x + 2 = 180
8x - 6 = 180
+6 +6
$\frac{8x}{8} = \frac{186}{8}$
$\boxed{x = 23.25}$

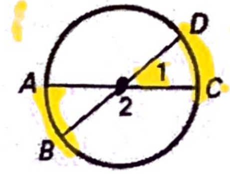
Angles Formed INSIDE OR OUTSIDE of a Circle

INSIDERS

Chords Intersect Inside the Circle/Angles Inside the Circle
If two chords intersect *inside* a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.

$$m\angle 1 = \frac{m\widehat{DC} + \widehat{AB}}{2}$$

$\frac{1}{2}$(sum of arcs) = angle

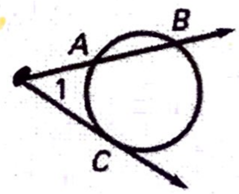


OUTSIDERS!

One Secant & One Tangent/Angles Outside the Circle
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

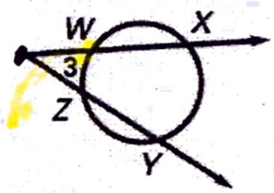
$$m\angle 1 = \frac{m\widehat{BC} - \widehat{AC}}{2}$$

angle = $\frac{1}{2}$(BIG ARC − small arc)



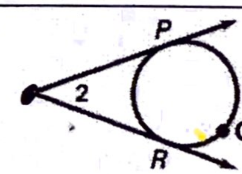
Two Secants/Angles Outside the Circle
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs

$$m\angle 3 = \frac{m\widehat{XY} - \widehat{WZ}}{2}$$



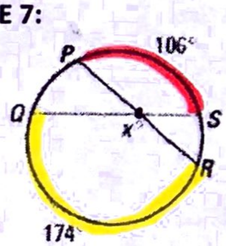
Two Tangents/Angles Outside the Circle
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

$$m\angle 2 = \frac{m\widehat{PQR} - \widehat{PR}}{2} \quad \underline{or} \quad m\angle 2 + m\widehat{PR} = 180$$



EXAMPLE 7:

a.



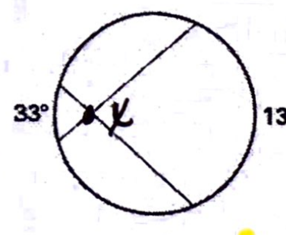
'insider'

$x = \frac{1}{2}(106+174)$

$x = \frac{1}{2}(280)$

$\boxed{x = 140}$

b.

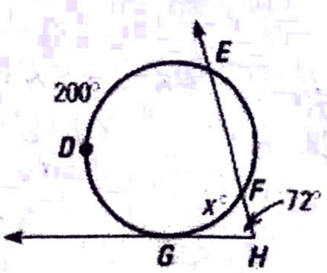


'insider'

$x = \frac{1}{2}(131+33)$

$\boxed{x = (82)}$

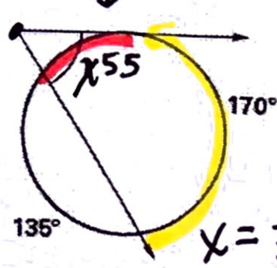
c.



d.

360 − 170 − 135 = 55

'outsider'



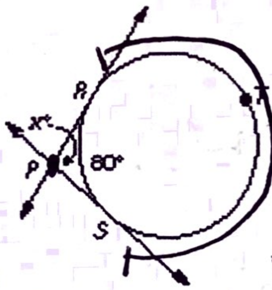
$x = \frac{1}{2}(170-55)$

$= \frac{1}{2}(115)$

$\boxed{x = 57.5}$

'outsider'

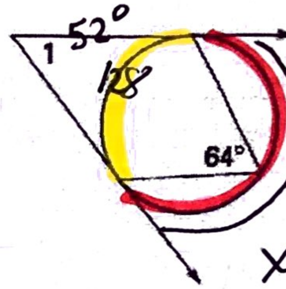
e.



$x = \frac{1}{2}(280-80)$

$= \frac{1}{2}(200) = 100$

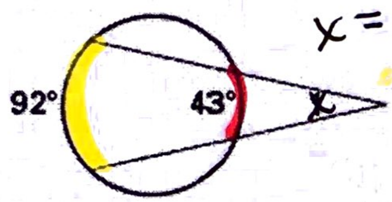
f.



$x = \frac{1}{2}(232-128)$

$\frac{1}{2}(104) = 52$

G.

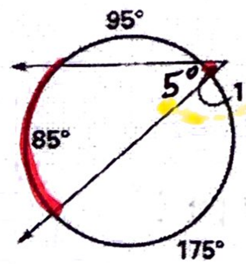


$x = \frac{1}{2}(92-43)$

$\frac{1}{2}(49)$

$\boxed{24.5}$

h.

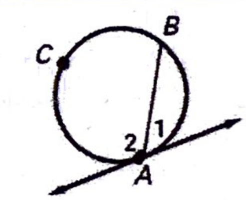


$x = \frac{1}{2}(85-5)$

$= \frac{1}{2}(80)$

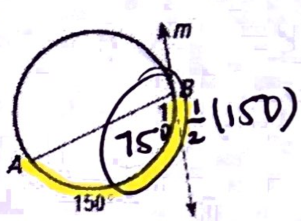
$= \boxed{40}$

<u>Other Angle Relationships in Circles</u> P.9

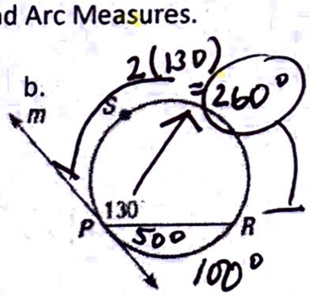
Tangent and Chord RULE: If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc. ∡1, ∡2 are inscribed!	

EXAMPLE 6: Finding Angle and Arc Measures.

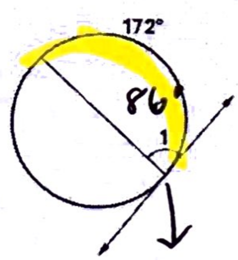
a.



b.

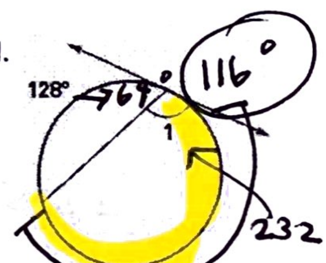


c.



$\frac{172}{2}$

d.



360-128=232