RATIONAL ROOT THEOREM

FACTOR & SOLVE: $6x^2 - 7x + 2 = 0$

$(3x-2)(2x-1) = 0$

$x = \frac{2}{3}, \frac{1}{2}$

$12x^2 - x - 6 = 0$

$(4x-3)(3x+2) = 0$

$x = \frac{3}{4}, \frac{-2}{3}$

RATIONAL ROOT THM:

POSSIBLE RATIONAL ROOTS $=$ $\frac{\text{factors of constant term}}{\text{factors of leading coefficient}}$

List all possible rational roots:

(1) $3x^3 - 3x^2 + 2x - 8 = 0$

$\frac{\pm(1, 2, 4, 8)}{\pm(1, 3)} = \pm\left(1, 2, 4, 8, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}\right)$

SOLVE.

(2) $x^3 - 7x + 6 = 0$

FACTOR?
NO GIVEN ROOT → USE RATIL THM!

possible roots $= \frac{\pm(1, 2, 3, 6)}{\pm(1)} = \pm(1, 2, 3, 6)$

2	1	0	-7	6
		2	4	-6
	1	2	-3	0

$x^2 + 2x - 3 = 0$

$(x - 1)(x + 3) = 0$

$x = 1, -3, 2$

(3) $4x^4 + 4x^3 + 17x^2 + 16x + 4 = 0$

possible roots $= \frac{\pm(1, 2, 4)}{\pm(1, 2, 4)} = \pm(1, 2, 4, \frac{1}{2}, \frac{1}{4})$

$x = -\frac{1}{2}$ DR, $\pm 2i$

$-\frac{1}{2}$	4	4	17	16	4
		-2	-1	-8	-4
	$4x^3$	2	16	8	0

FACTOR $(4x^3 + 2x^2 + 16x + 8) = 0$

$2x^2(2x + 1) + 8(2x + 1) = 0$

$(2x + 1)(2x^2 + 8) = 0$

$2x + 1 = 0$

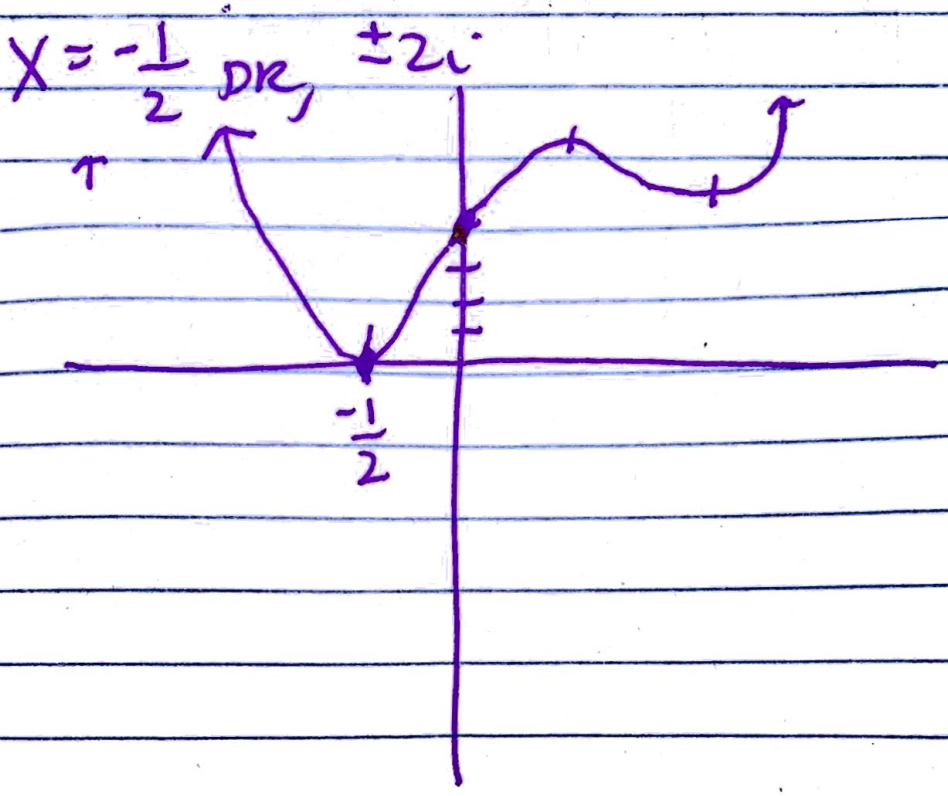
$x = -\frac{1}{2}$

$2x^2 + 8 = 0$

$2x^2 = -8$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$


$x = -\frac{1}{2}$ DR
$\pm 2i$
$-\frac{1}{2}$

② Solve: $x^3 - 7x + 6 = 0$

possible rat'l roots: $\frac{\pm(1,2,3,6)}{\pm 1} = \pm(1,2,3,6)$

1 | 1 0 -7 6
↓ 1 1 -6
1 1 -6 | 0

$x^2 + x - 6 = 0$
$(x-2)(x+3) = 0$
$\boxed{x = 2, -3, 1}$

③ $4x^4 + 4x^3 + 17x^2 + 16x + 4 = 0$

$\frac{\pm(1,2,4)}{\pm(1,2,4)} = \pm(1, 2, 4, \frac{1}{2}, \frac{1}{4})$

~~-1~~ 4 4 17 16 4
-4 0 -17 1
4 0 17 -1 | 5

$x^2 = -4$

$x^2(2x+1) + 4(2x+1)$
$(4x^3 + 2x^2 + 16x + 8) = 0$
$2(2x^3 + x^2 + 8x + 4) = 0$
$2x^2(2x+1) + 8(2x+1) = 0$
$(2x+1)(2x^2+8) = 0$

1 | 4 4 17 16 4
2 9 10 13
4 8 20 26 17

$-\frac{1}{2}$ | 4 4 17 16 4
-2 -1 -8 -4
4 2 16 8 | 0

$\boxed{x = -\frac{1}{2} \text{ DR}, \pm 2i}$

Honors Math 3

More 8.5/8.6

1. Solve $x^4 - 6x^3 + 6x^2 + 24x - 40 = 0$, given that $3 + i$ is a root.

conjugate $3-i$

Sum: $3+i+3-i=6$

product: $(3+i)(3-i) = 9 - i^2 = 9 - -1 = 10$ $\Big\}\ x^2 - 6x + 10$

$$x^2 - 6x + 10 \overline{)\ x^4 - 6x^3 + 6x^2 + 24x - 40}$$

x^2 $\quad -4$

$x^2(x^2 - 6x + 10)$ $(+)\ -x^4 + 6x^3 - 10x^2$

$-4x^2 + 24x - 40$

$-4(x^2 - 6x + 10)$ $(+)\ +4x^2 - 24x + 40$

0

$\frac{x^4}{x^2}$ $\quad \frac{-4x^2}{x^2}$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = 2, -2$

$\boxed{x = 3 \pm i,\ 2,\ -2}$

2. Find a ~~cubic~~ equation with real coefficients that has the following roots:

a) $-1, 3-i, 3+i$

$(x+1)$

sum: 6

prod.: 10 $\Big\}\ (x^2 - 6x + 10)(x+1)$

$x^3 + x^2 - 6x^2 - 6x + 10x + 10$

$\boxed{x^3 - 5x^2 + 4x + 10 = 0}$

b) $\sqrt{2}, i\sqrt{5}, -\sqrt{2}, -i\sqrt{5}$

sum: $i\sqrt{5} + -i\sqrt{5} = 0$

prod. $(i\sqrt{5})(-i\sqrt{5}) = -i^2 \cdot 5 = -(-1) \cdot 5 = 5$ $\Big\}\ (x^2 + 5)$

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sum: $\sqrt{2} + -\sqrt{2} = 0$

prod.: $(\sqrt{2})(-\sqrt{2}) = -2$ $\Big\}\ (x^2 - 2)(x^2 + 5)$

$\boxed{x^4 + 3x^2 - 10 = 0}$