Math 3 Unit 2: Guided Notes for the Remainder Theorem

Use either synthetic or long division to find the quotient of the following example.

$(x^4 + 0x^3 + 3x^2 + 4x - 1) \div (x - 1)$ Quotient $x^3 + x^2 + 4x + 8 + \frac{7}{x-1}$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & 3 & 4 & -1 \\ & & +1 & +1 & +4 & +8 \\ \hline & 1 & 1 & 4 & 8 & 7 \end{array}$$

Remainder 7

Next evaluate the function $f(x) = x^4 + 3x^2 + 4x - 1$ at $f(1)$. 7

What do you notice? $1^4 + 3(1)^2 + 4(1) - 1 =$

When dividing $f(x)$ by $(x-1)$ the remainder is equal to $f(1)$.

The Remainder theorem tells us that we can find the value of a polynomial function for some value, c, by using long or synthetic division. When we divide $f(x)$ by $x - c$, the f(c) will equal the remainder.

Example 1: Given $f(x) = -x^3 + 3x^2 + 0x - 5$, find $f(5)$ using synthetic ~~or long division.~~

shelf number

$$\begin{array}{r|rrrr} 5 & -1 & 3 & 0 & -5 \\ & \downarrow & +-5 & +-10 & +-50 \\ \hline & -1 & -2 & -10 & -55 \end{array}$$

$\boxed{f(5) = -55}$

Example 2: Given $f(x) = x^3 - x^2 - x + 4$, find $f(-3)$ using synthetic ~~or long division.~~

$$\begin{array}{r|rrrr} -3 & 1 & -1 & -1 & 4 \\ & \downarrow & +-3 & +12 & +-33 \\ \hline & 1 & -4 & 11 & -29 \end{array}$$

$\boxed{f(-3) = -29}$

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Example 3: Given $f(x) = x^3 + 4x^2 - x - 4$, find $f(x)$ for x = −4 using synthetic or long division.

$$\begin{array}{r|rrrr} -4 & 1 & 4 & -1 & -4 \\ & & -4 & 0 & 4 \\ \hline & 1x^2 & +0x & -1 & 0 \end{array}$$

$\boxed{f(-4) = 0}$

Knowing that the reminder is zero in the last example is very useful!!

It tells us that $(x + 4)$ is a factor of $f(x) = x^3 + 4x^2 - x - 4$ We can now use this to find the remaining factors and roots of $f(x)$. SOLVE

$x^2 - 1 = 0$

$+1 \quad +1$

$\sqrt{x^2} = \sqrt{1}$

$\boxed{x = \pm 1, -4}$

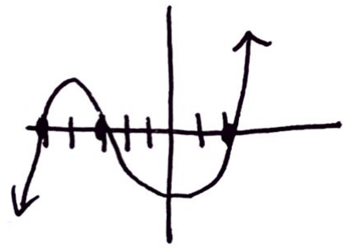
roots / x-int. / solutions

Example 1: Find the remaining factors of $f(x) = x^3 + 6x^2 - x - 30$ given one factor is $(x + 5)$. Graph f(x).

$=0$

$x=-5$

$$\begin{array}{r|rrrr} -5 & 1 & 6 & -1 & -30 \\ & & -5 & -5 & 30 \\ \hline & 1 & 1 & -6 & \vert\ 0\ \checkmark \end{array}$$



$$x^2 + x - 6 = (x-2)(x+3) = 0$$

$x-2=0$

$x+3=0$

$$\boxed{x = 2, -3, -5}$$

Example 2: Find the remaining factors of $f(x) = x^3 - 11x^2 + 36x - 36$ given a factor is $(x - 6)$. Graph f(x).

$=0$

$x=6$

$$\begin{array}{r|rrrr} 6 & 1 & -11 & 36 & -36 \\ & & 6 & -30 & 36 \\ \hline & 1 & -5 & 6 & \vert\ 0\ \checkmark \end{array}$$

$$x^2 - 5x + 6 = (x-3)(x-2)$$

$x-3=0$

$x-2=0$

$$\boxed{x = 2, 3, 6}$$

Example 3: Find the remaining factors of $f(x) = -x^3 - 4x^2 + 12x + 48$ given a factor of $(x + 4)$. Graph f(x).

$$\begin{array}{r|rrrr} -4 & -1 & -4 & 12 & 48 \\ & & 4 & 0 & -48 \\ \hline & -1 & 0 & 12 & \vert\ 0 \end{array}$$

$$-\cancel{x^2} + 12 = 0$$
$$+\cancel{x^2} \qquad +x^2$$

$$\sqrt{x^2} = \sqrt{12} < \begin{matrix} 4 \\ 3 \end{matrix}$$

$$\boxed{x = \pm 2\sqrt{3}, -4}$$

$x=3$

Example 4: Find the remaining factors of $f(x) = x^4 - 3x^3 + 27x - 81$ given two factors; $(x - 3)$ and $(x + 3)$. Graph f(x).

$x=-3$

$$\begin{array}{r|rrrrr} 3 & 1 & -3 & 0 & 27 & -81 \\ & & 3 & 0 & 0 & 81 \\ \hline -3 & 1 & 0 & 0 & 27 & \vert\ 0\ \checkmark \\ & \downarrow & -3 & 9 & -27 & \\ \hline & 1x^2 & -3x & 9 & \vert\ 0 & \end{array}$$

$$x^2 - 3x + 9 = 0$$

$a=1,\ b=-3,\ c=9$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

2 imag. root

$$x = \frac{3 \pm \sqrt{-27}}{2} = \boxed{\frac{3 \pm i\sqrt{27}}{2}}$$

$3, -3$

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